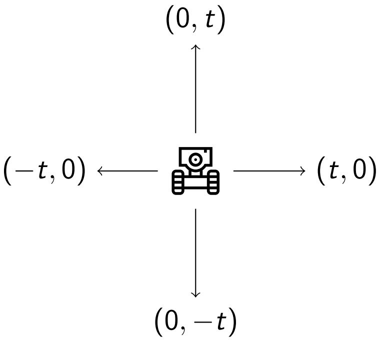
Motion planning in known environment

Viktória Vozárová

December 9, 2018

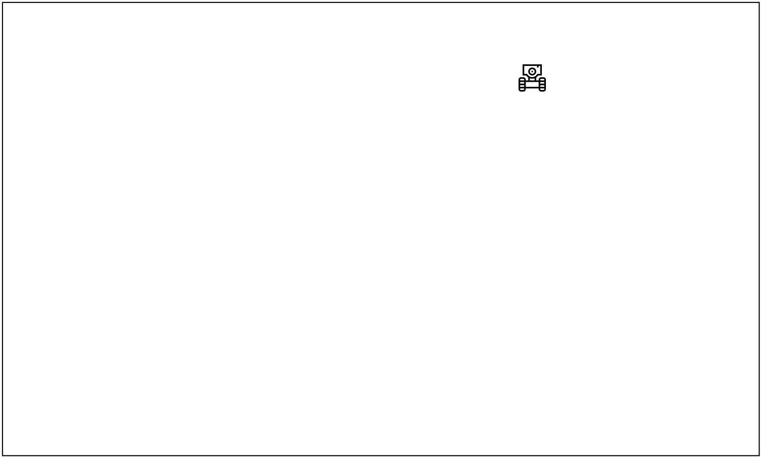
Example – model

- $\mathbb{R}^2$ space with obstacles
- restricted robot movements
- the robot is **nonholonomic**



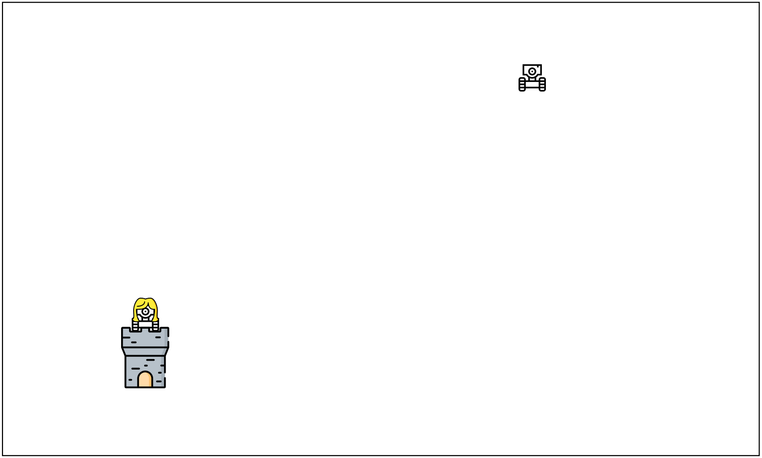
Example – problem

The robot is set in a metric space X on initial position x_{init}.



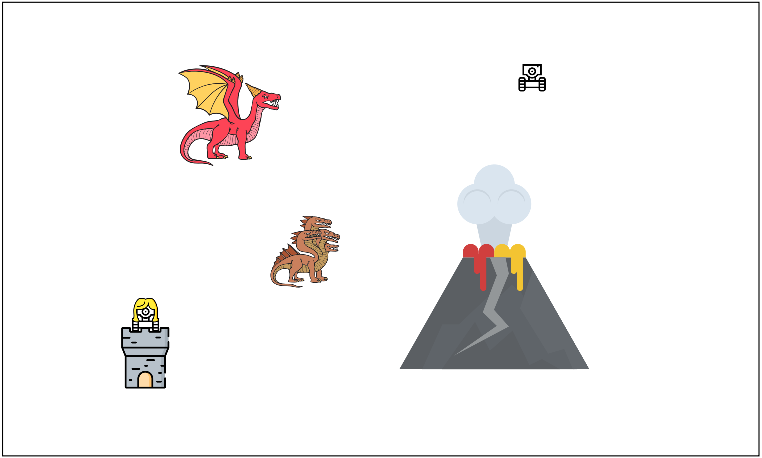
Example – problem

His objective is to reach the roboprincess in $X_{goal} \subset X$.



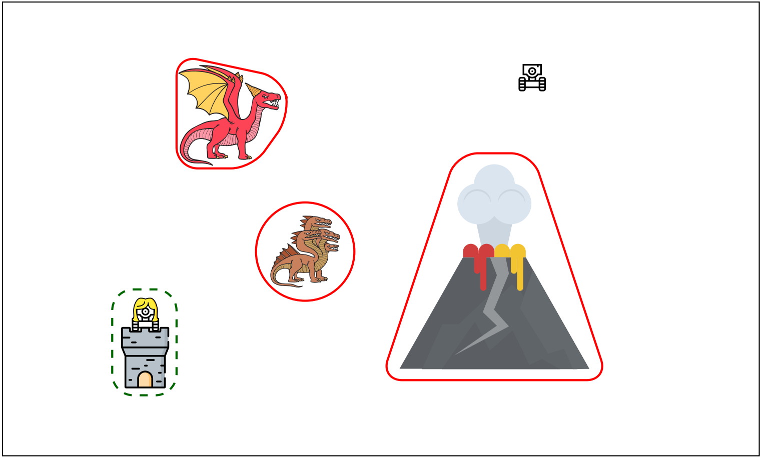
Example – problem

There are some obstacles on the way $X_{obs} \subset X$.



Example – problem

Let's make it more formal.



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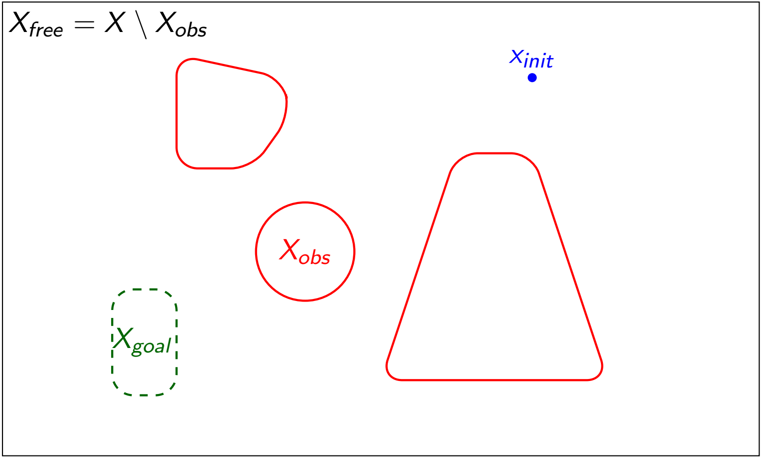

RRT – Rapidly-exploring random tree

Initialize tree $T = (\{x_{init}\}, \emptyset)$.

[1] d is a metric of X

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4. Add the vertex x_{new} and the edge (x_{near}, x_{new}) to T.

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5. Repeat until $x_{new} \in X_{goal}$.

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Return path from x_{init} to $x \in X_{goal}$.

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RRT*

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1. Pick random $x_{rand} \in X_{free}$.
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3. Find $x_{new} \in X_{free}$ as in RRT.

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4. Find x_{parent} in T such that:

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4. Find x_{parent} in T such that:
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5. Add the vertex x_{new} and the edge (x_{parent}, x_{new}) to T.
6. For each x in T where $d(x, x_{new}) < \varepsilon$:

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5. Add the vertex x_{new} and the edge (x_{parent}, x_{new}) to T.
6. For each x in T where $d(x, x_{new}) < \varepsilon$:
   - if the path from x_{init} to x can be improved by changing x's parent to x_{new}, rewire.

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7. Repeat until $x_{new} \in X_{goal}$.

Return path from x_{init} to $x \in X_{goal}$.

---

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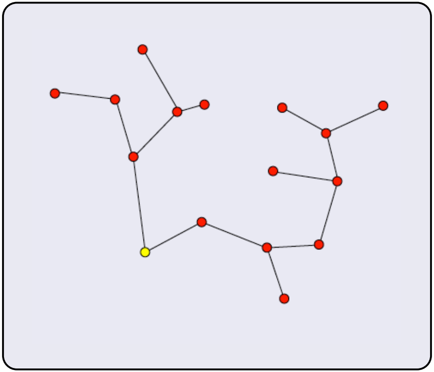
PRM, RRT, and RRT*

Frank Dellaert

Based on materials by Steve Lavalle, Lydia Kavraki, Emilio Frazzoli and their students

RRT*



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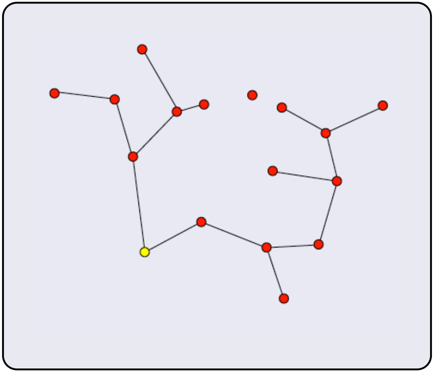

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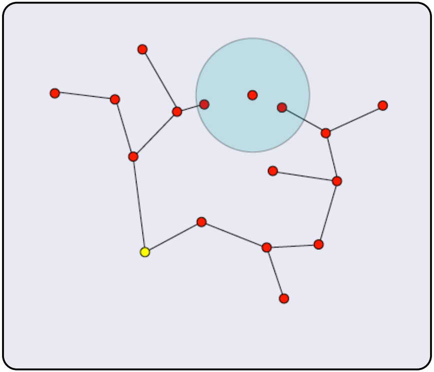

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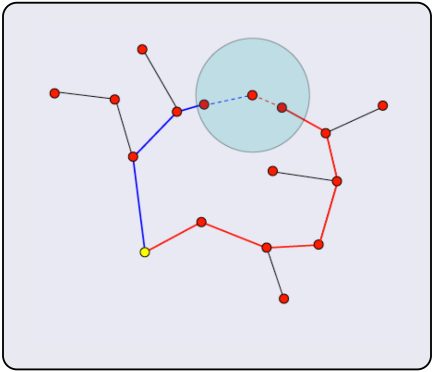

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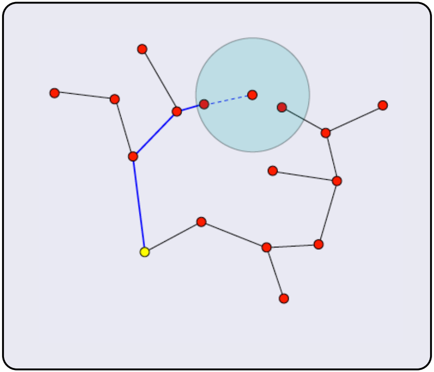

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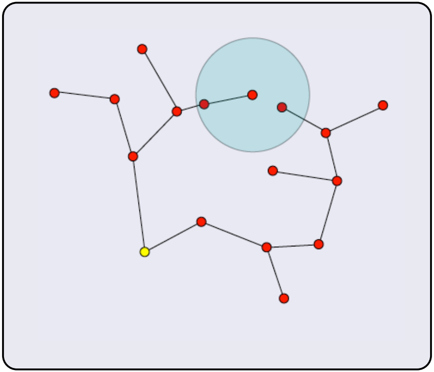

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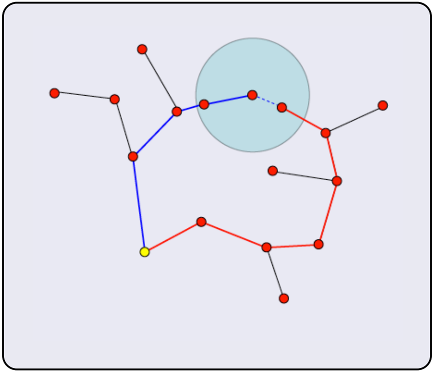

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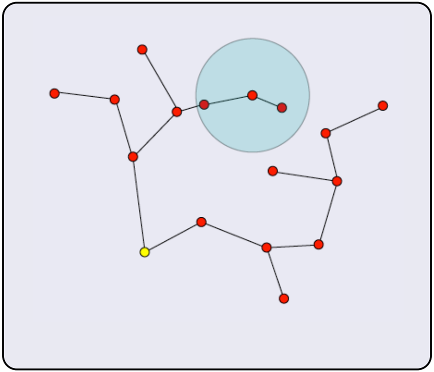

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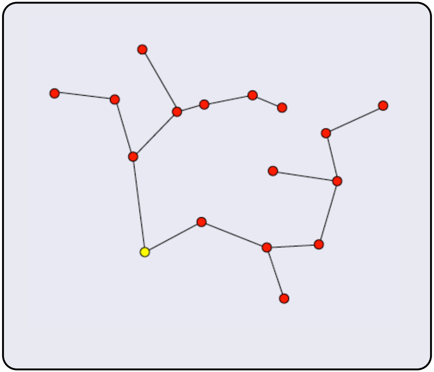
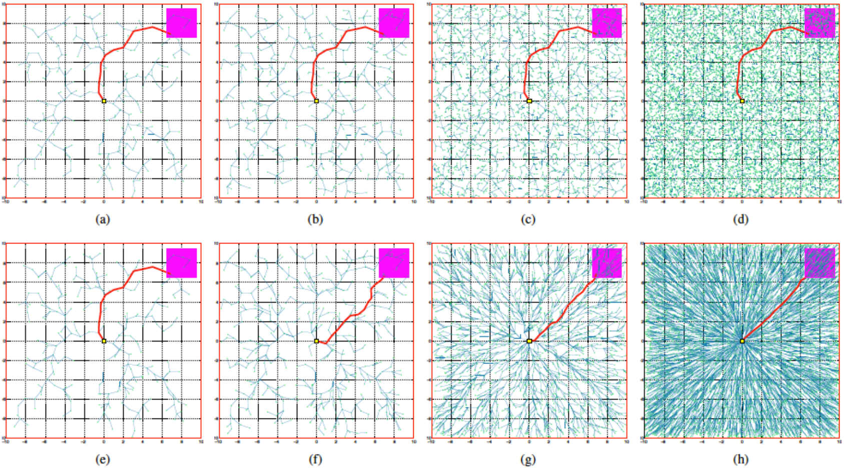


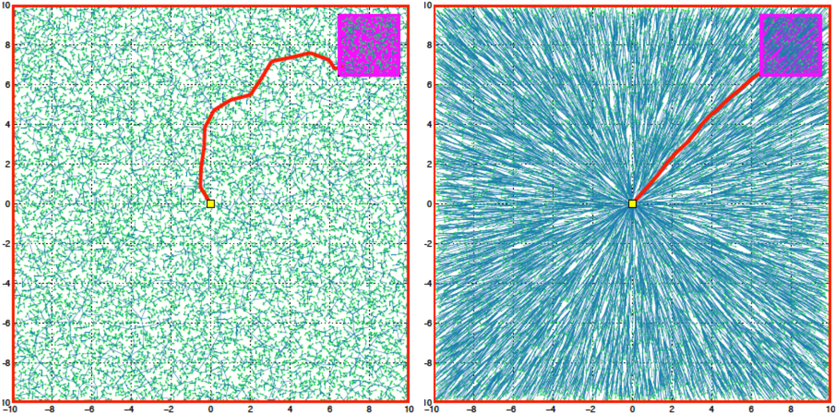
RRT vs RRT*



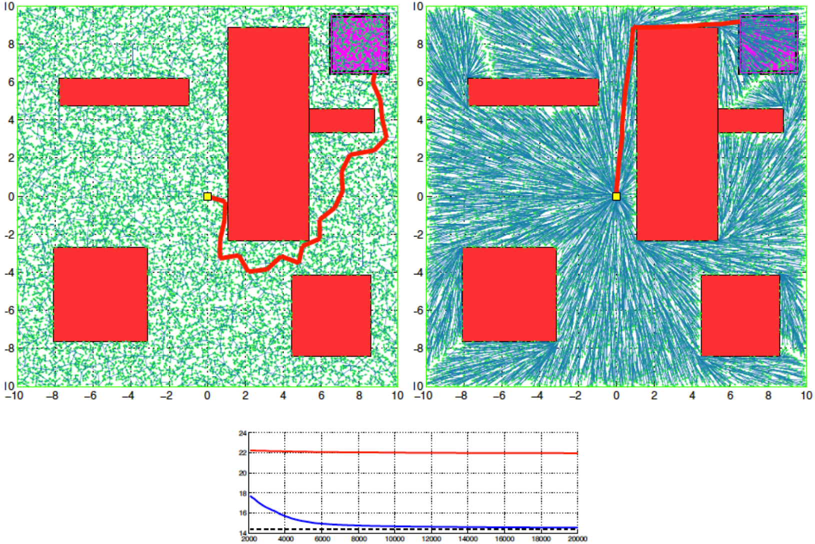
(a) (b) (c) (d)

(e) (f) (g) (h)

RRT vs RRT*



RRT* with Obstacles



Anytime RRT*

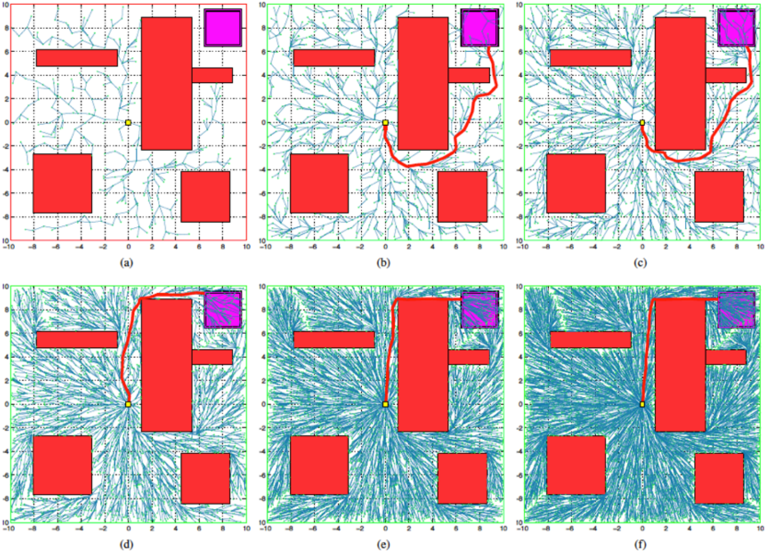


Fig. 4. RRT* algorithm shown after 500 (a), 1,500 (b), 2,500 (c), 5,000 (d), 10,000 (e), 15,000 (f) iterations.

Other heuristics

- Bidirectional RRT
- Rapidly-exploring random graph (RRG)
- Informed RRT*
- RRT*-Smart
- Real-Time RRT*