Adapting Biochemical Kripke Structures for Distributed Model Checking

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Content

1. Bounded Hamming Distance Kripke Structures
2. Distributed model checking
3. Fragmenting Kripke structures

Model checking on chemical reactions

- Reactants $\rightarrow$ products
- Non-deterministic

$A + B$	$\rightarrow$	$B + C$
$A + B + \neg C$	$\rightarrow$	$\neg A + B + C$
$A + B + \neg C$	$\rightarrow$	$A + B + C$
$A + B + C$	$\rightarrow$	$\neg A + B + C$
$A + B + C$	$\rightarrow$	$A + B + C$

- Few entities on each side (maximum is 6 in all public databases)

The resulting Kripke Structure

$M = (S, I, R, \mathcal{L})$ is a k - Bounded Hamming Distance Kripke Structure (k-BHDKS) when:

$\forall s, s' \in S, \quad R(s, s') \Rightarrow (H(\mathcal{L}(s), \mathcal{L}(s')) \leq k)$

Properties:

- Relatively sparse: each state has at most $|\mathcal{AP}|^k$ neighbours
- Can be effectively distributed into fragments

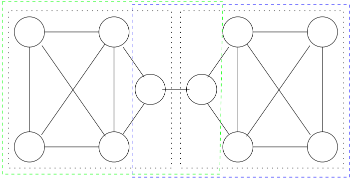
Assumption based model checking

A Kripke Structure is decomposed into fragments

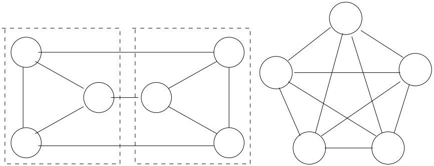
Each distributed node stores only one fragment, thus allowing to process larger model checking problems

A fragment is created by extending a subset of the state-space to its immediate neighbors

Kripke structure fragments



Obrázek: Dividing a KS in two fragments



Obrázek: KS with no possible fragmentation

Distributed fragment

A distributed fragment M' of $M = (S, R)$ around a set of core states $T \subseteq S$:

$M' = (S_T, R_T)$
$S_T = \{s \in S | s \in T \vee \exists s' \in T \textit{ such that } (s', s) \in R\}$
$R_T = \{(s_1, s_2) \in R | s_1 \in T, s_2 \in S_T\}$

Separator V of a set T is the minimal set of states, satisfying that there is no path from T to $S \setminus T$ avoiding all states in V

Existence of fragmentation in *k*-BHDKS

- Let $|T|$ be the size of the core set
- Each state in a *k*-BHDKS has at most $|\mathcal{AP}|^k$ neighbors
- The size of the fragment is at most $|T| + |T|.|\mathcal{AP}|^k$ - grows polynomially with the number of propositions

A hypercube

- A k-BHDKS with $|\mathcal{AP}|$ propositions can be partitioned to an l-dimensional hypercube as long as $l < |\mathcal{AP}|/k$
- Construction of the partitioning:
  - Create 2^l symmetricaly placed centers a_0 to a_{2^l-1} as states $0^{l.p}$, $0^{(l-1).p}1^p, \ldots, 1^{l.p}$ where p is $\left\lceil \frac{|\mathcal{AP}|}{l} \right\rceil$
  - Add a state s to fragment n if a_n is the nearest center from s with respect to Hamming distance
  - Include each fragment n include all immediate successors (the border)

Properties of the partitioning

- The size of the core is approx. $\frac{1}{2^l}.|KS|$
- The size of the border associated with the core is below $\frac{l}{2^l}.|KS|$
- Transition can exist only between states in adjacent nodes of the hypercube

The End

Thank you for your attention

Discussion