

# On Combining Partial Order Reduction with Fairness Assumptions

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- 1 Fairness
- 2 Partial Order Reduction
- 3 Behaviour subclasses
- 4 Results

*Fairness* is a restriction on set of behaviours

*Unconditional fairness*: **GF**  $executed_{\alpha}$

*Strong fairness*: **GF**  $enabled_{\alpha} \Rightarrow$  **GF**  $executed_{\alpha}$

*Wrong fairness*: **FG**  $enabled_{\alpha} \Rightarrow$  **GF**  $executed_{\alpha}$

# Partial Order Reduction

*Partial order reduction* is a technique for reducing the state space of asynchronous system model.

Conditions (dependence, visibility, ...) constraining the reduced state space (ample sets).

Equivalence (deadlock, stuttering, ...) partitions set of behaviours.

Each behaviour have its *representant* in the reduced state space.

**C0**  $ample(s) = \emptyset$  iff  $enabled(s) = \emptyset$ .

**C1** Along every path in the model starting from at  $s$ , the following condition holds: a transition that is *dependent* on a transition in  $ample(s)$  cannot occur without a transition in  $ample(s)$  occurring first.

**C2** If  $enabled(s) \neq ample(s)$ , then every  $\alpha \in ample(s)$  is *invisible*.

**C3** A cycle is not allowed if it contains a state in which some transition  $\alpha$  is enabled, but is never included in  $ample(s)$  for any state  $s$  on the cycle.

**C4** From every state  $s$  there is reachable a fully expanded state that is state  $s'$  such that  $ample(s') = enabled(s')$ .

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Paths with infinitely many visible actions

Process fair paths

Weakly fair paths

Strongly fair paths

# Counterexample – weakly fair paths

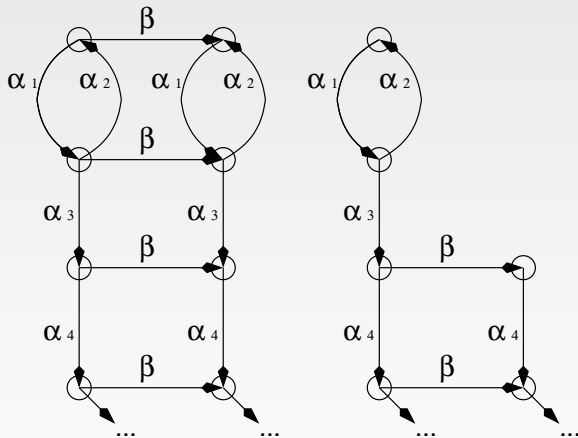


Figure: Model and its reduction

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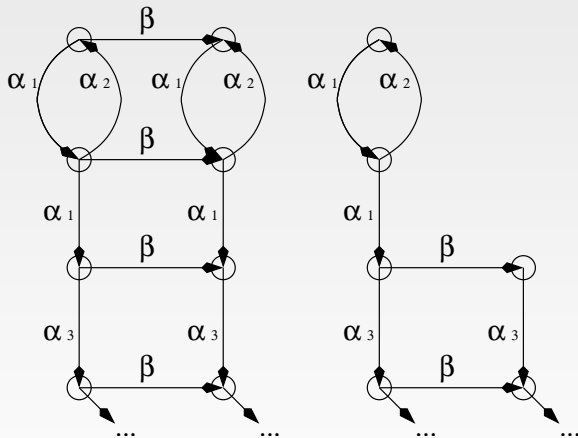


Figure: Model and its reduction

- 1 Certain subset of actions of modeled system is considered and each of them is taken infinitely many times.
- 2 Certain subset of processes of modeled multi-process system is considered and each of them performs an action infinitely many times.
- 3 Only strongly fair behaviours (paths) of modeled system are considered.
- 4 Only weakly fair behaviours (paths) of modeled system are considered.

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Better reduction

Effective way of checking validity

Spurious counterexamples

Not worth publishing?!