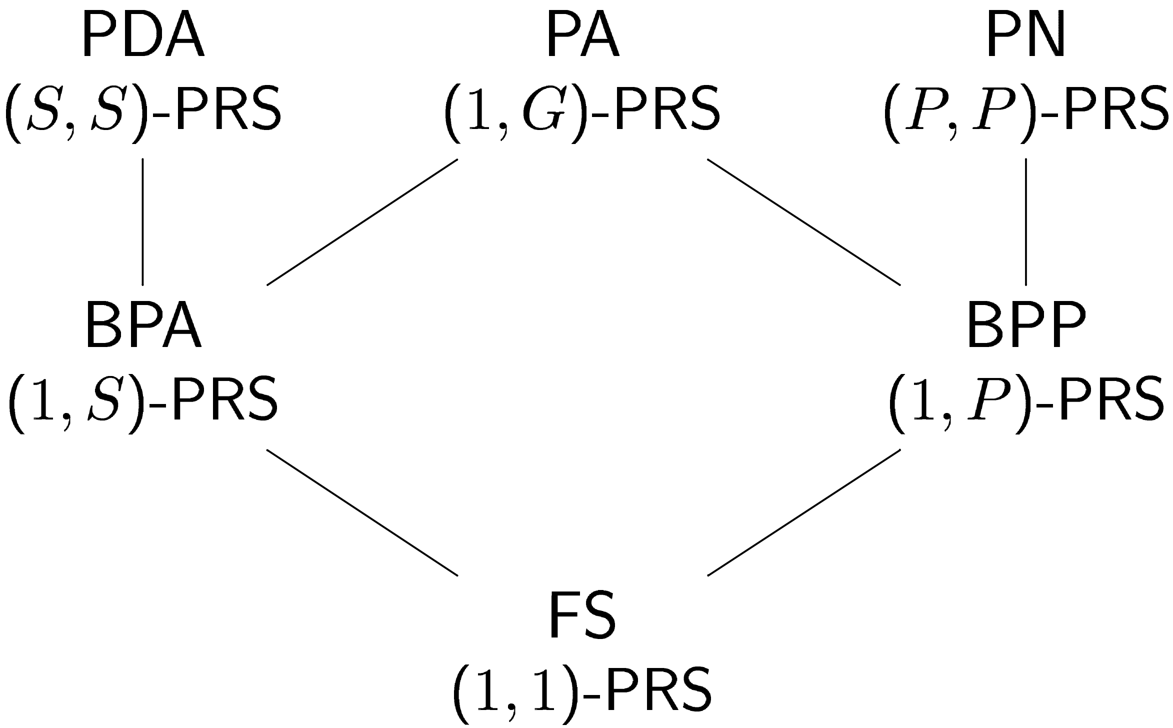
ParaDiSe 2006

Weakly Extended PRS

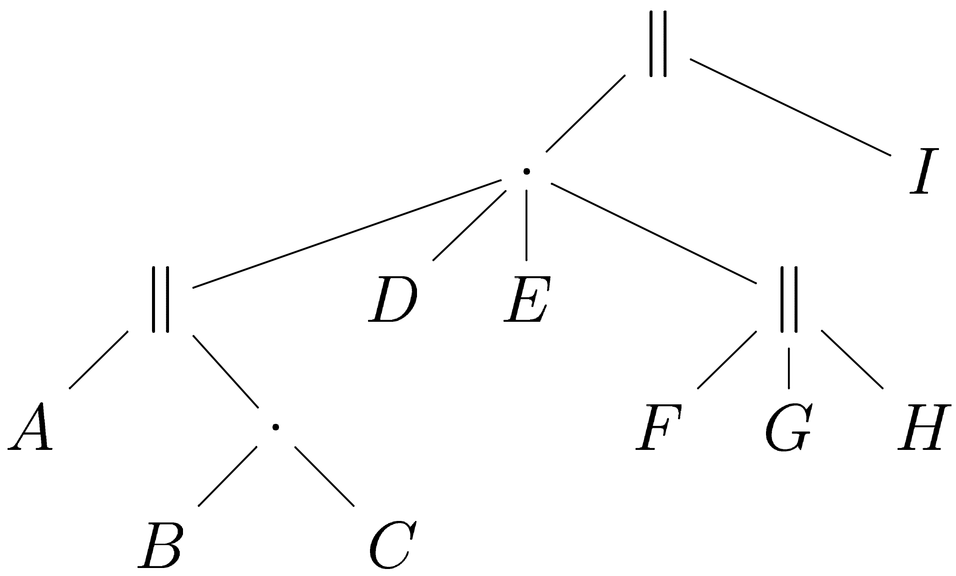
Mojmír Křetínský, Vojtěch Řehák, and Jan Strejček
Faculty of Informatics
Masaryk University

Bottom PRS-hierarchy



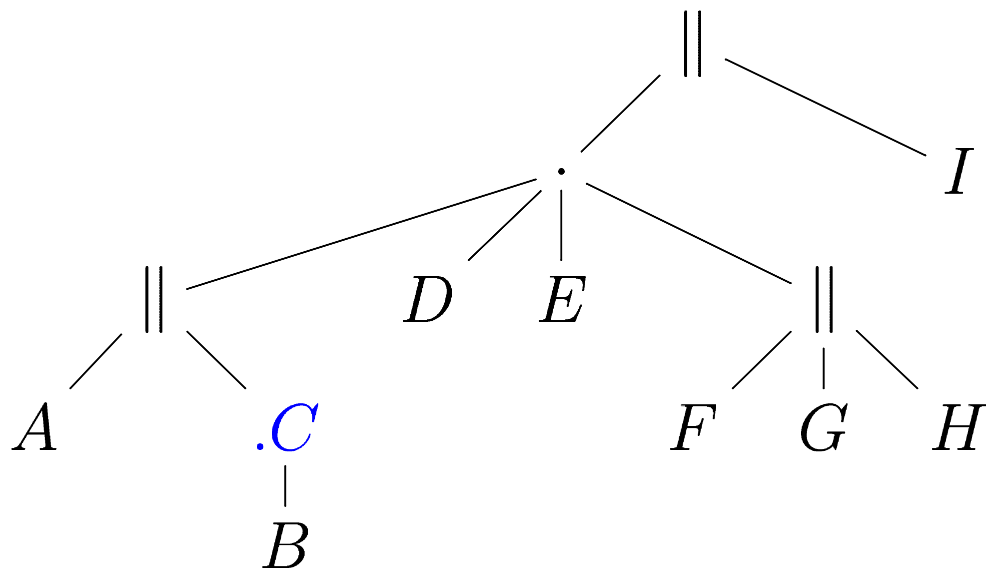
Term rewriting

$$((A\|(B.C)).D.E.(F\|G\|H))\|I$$



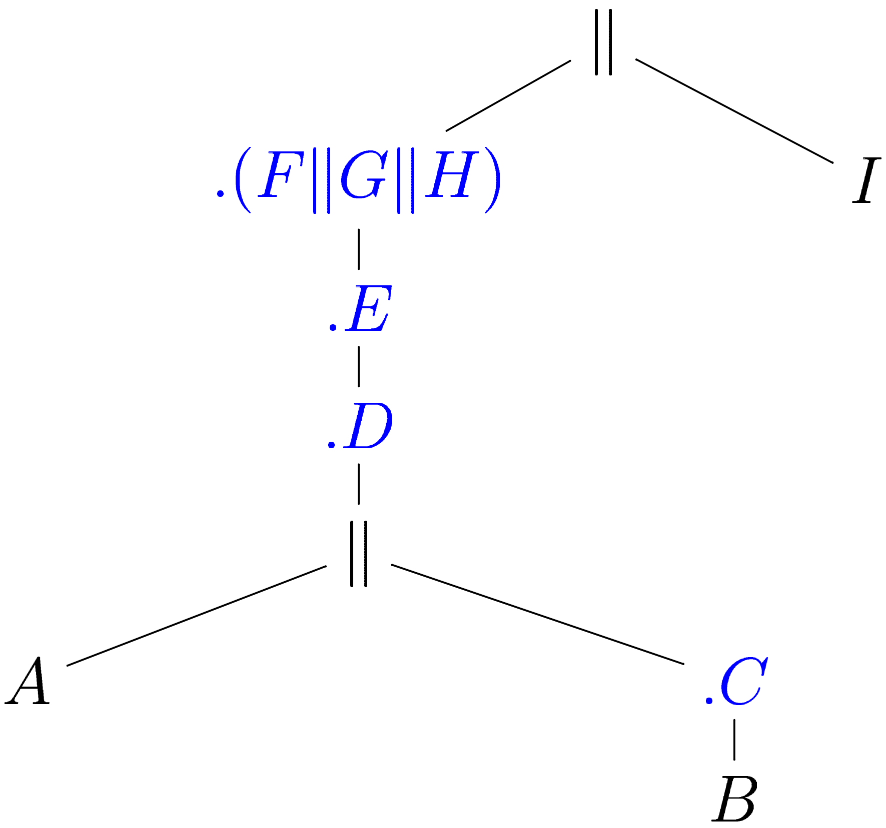
Term rewriting

$$((A\|(B.{\color{blue}C})).D.E.(F\|G\|H))\|I$$



Term rewriting

$$((A \| (B.C)).D.E.(F \| G \| H)) \| I$$



PRS-hierarchy

PRS
(G, G)-PRS

PAD
(S, G)-PRS

PAN
(P, G)-PRS

PDA
(S, S)-PRS

PA
$(1, G)$-PRS

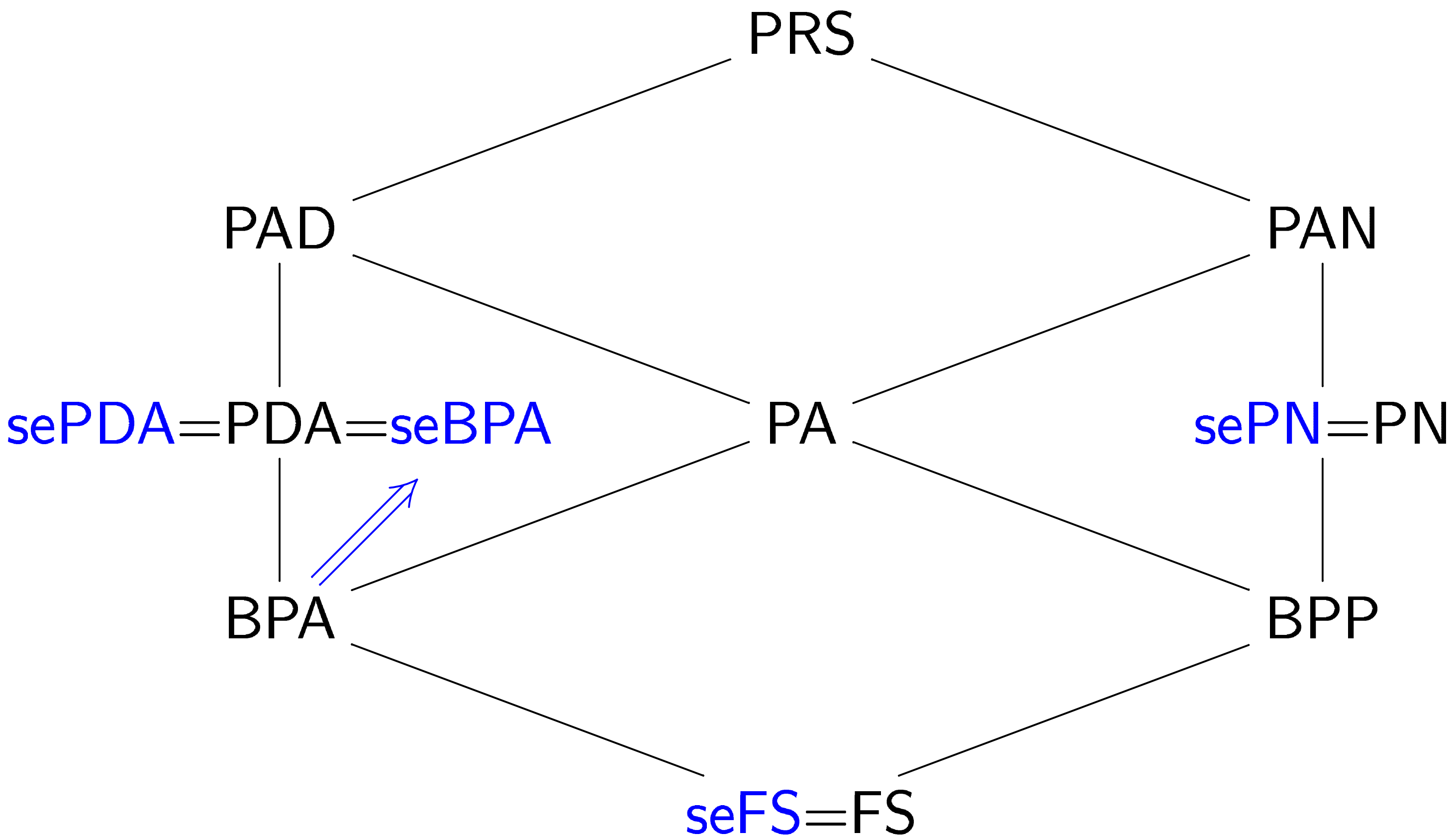
PN
(P, P)-PRS

BPA
$(1, S)$-PRS

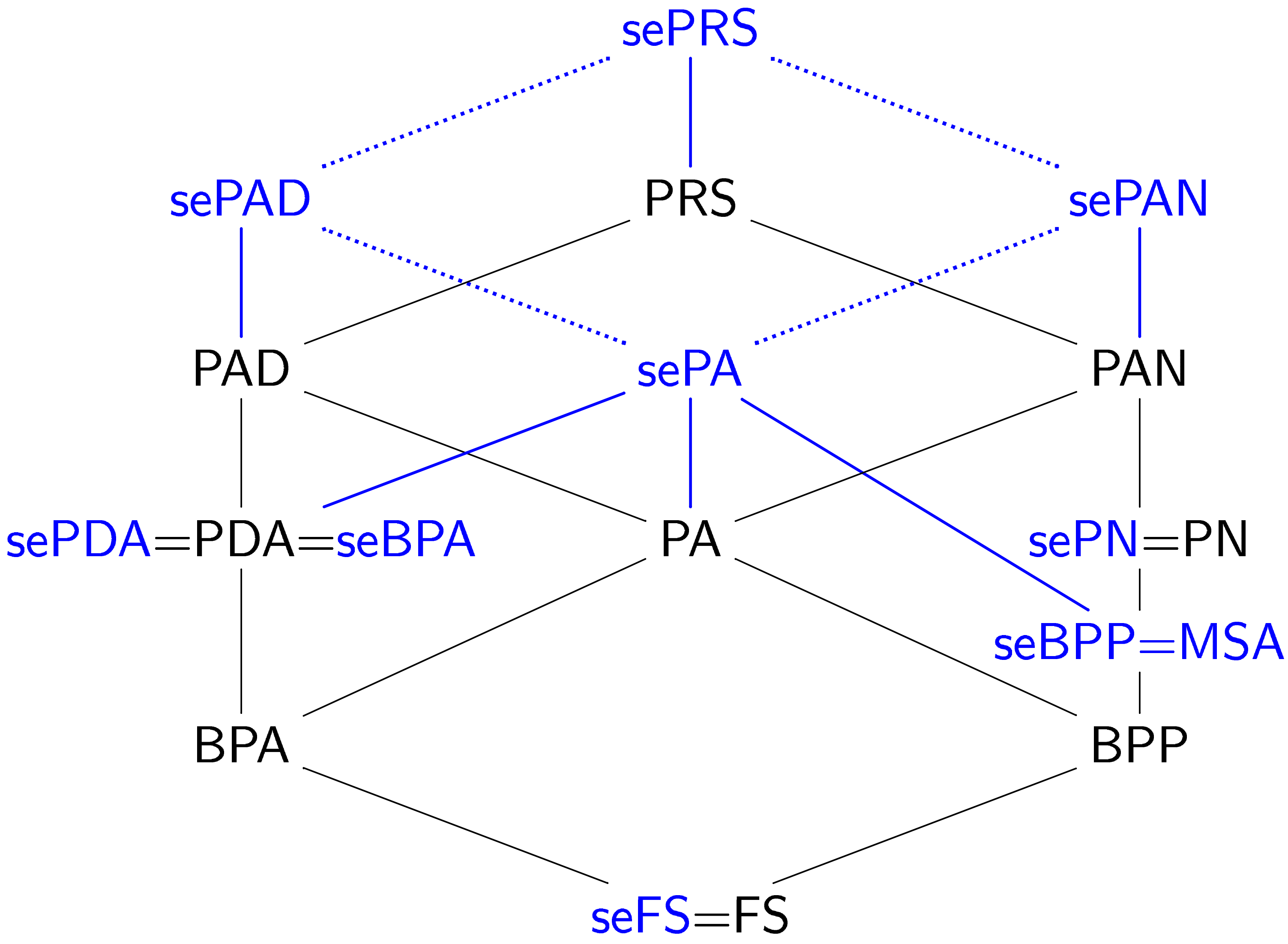
BPP
$(1, P)$-PRS

FS
$(1, 1)$-PRS

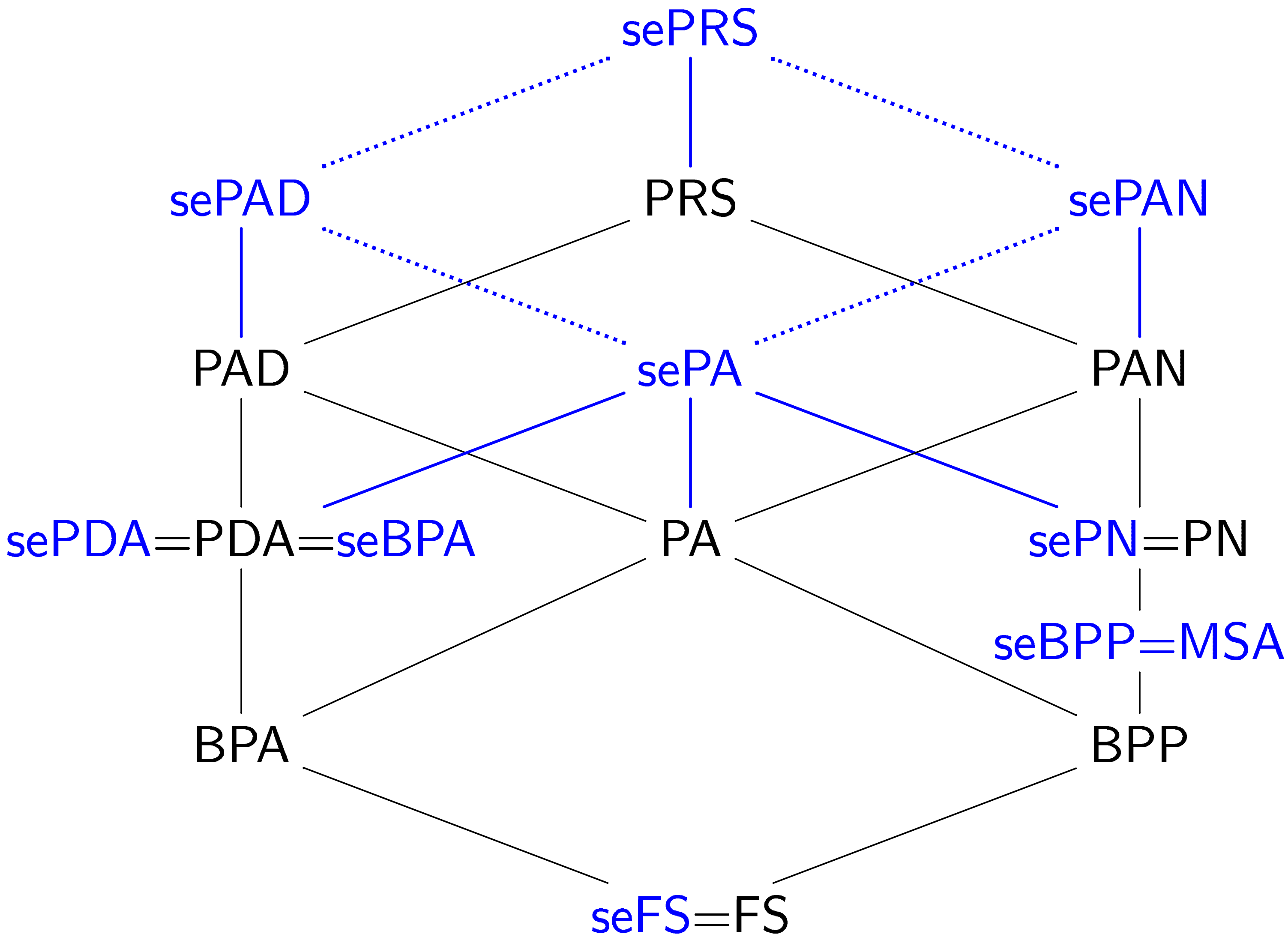
Coinciding State Extended Classes



State Extended PRS-hierarchy



State Extended PRS-hierarchy



Motivation for Weak State Extension

4 of 5 new sePRS classes have a full Turing-power

$\Downarrow$

sePRS are too strong

$\Downarrow$

PRS with weak finite-state unit [Infinity 2003]

1-weak (or very weak) restriction from the automata theory

Extending Units Overview

(α, β)-sePRS with finite-state unit

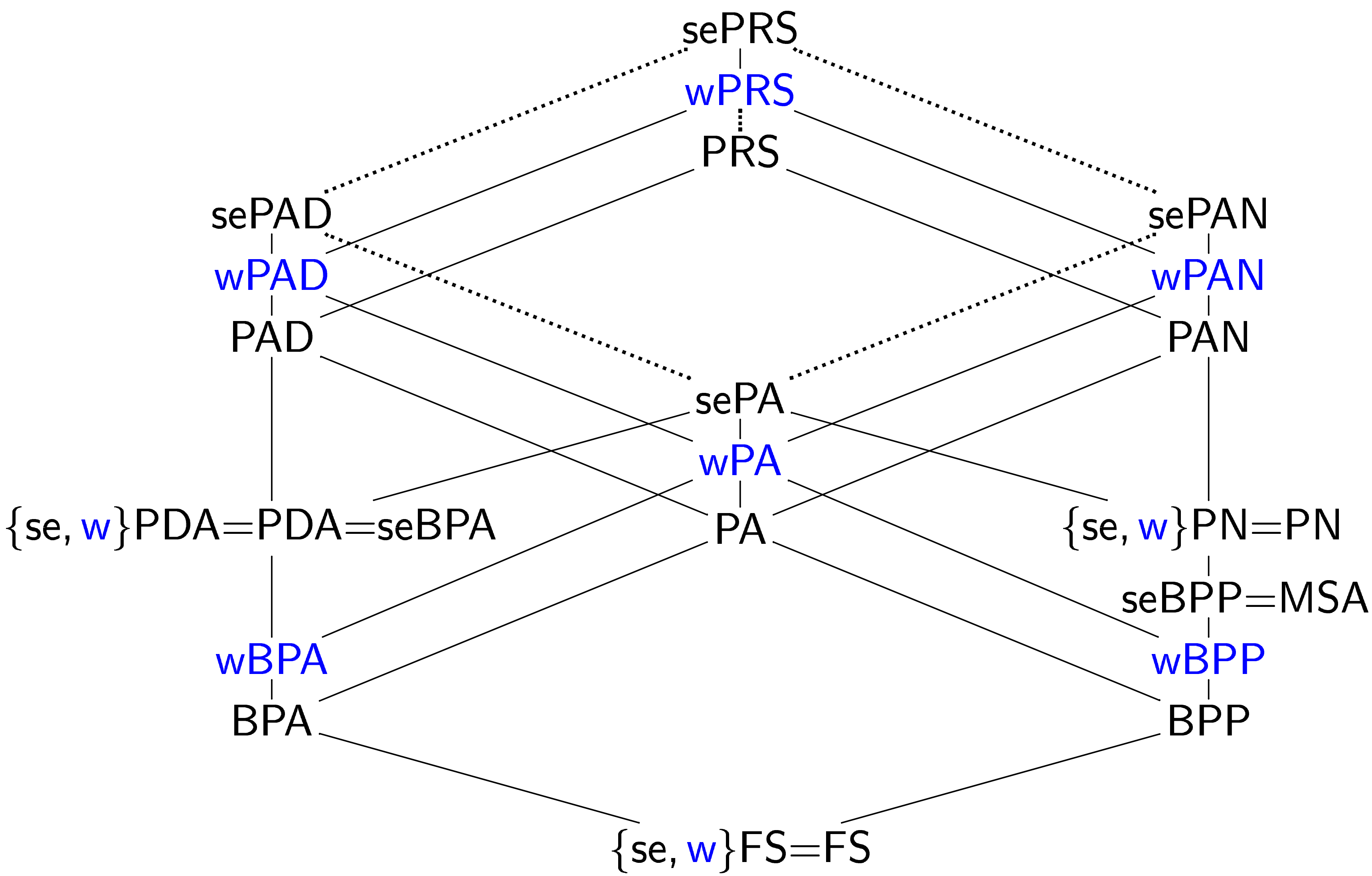
|

(α, β)-wPRS with weak finite-state unit

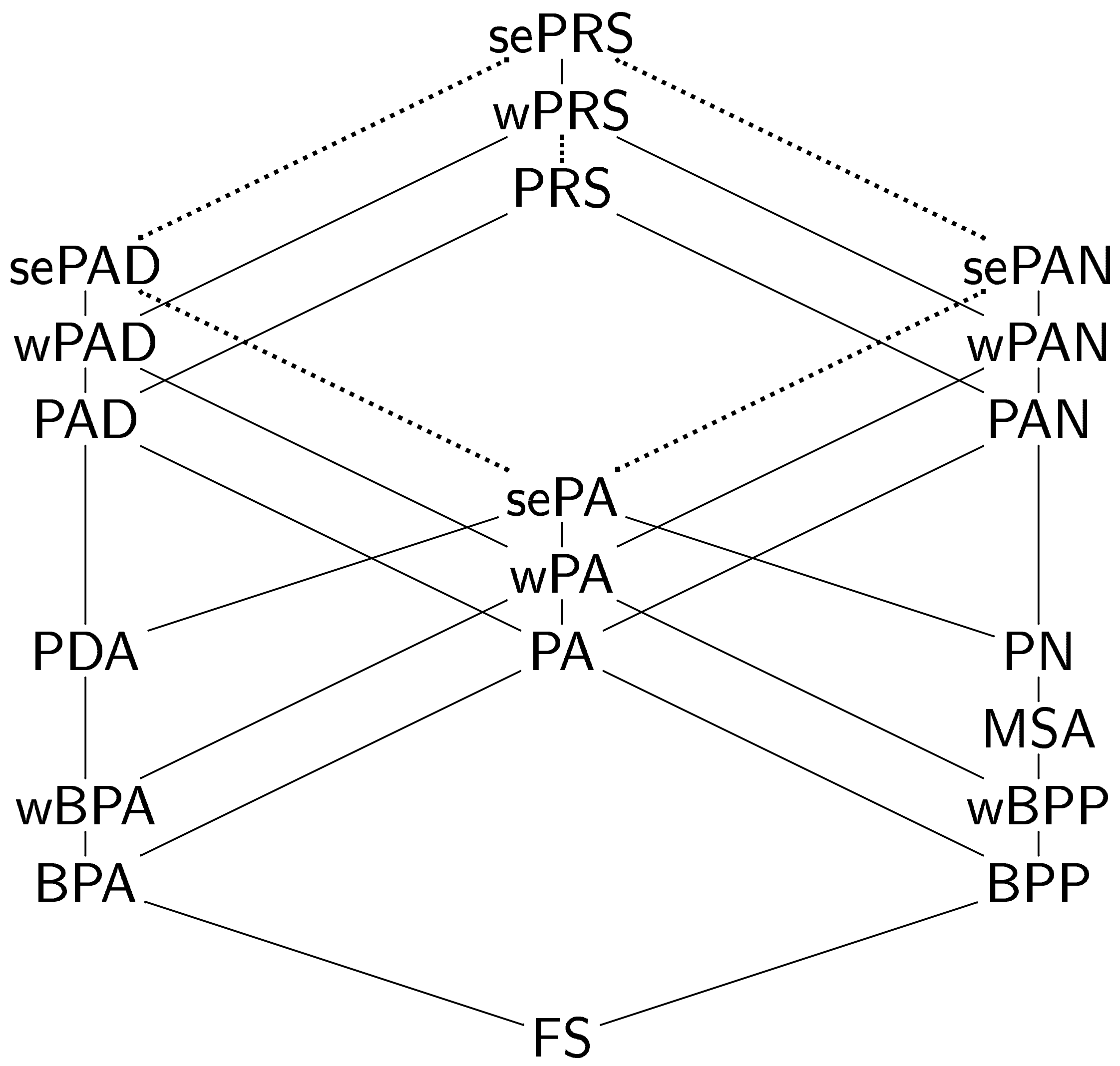
|

(α, β)-PRS

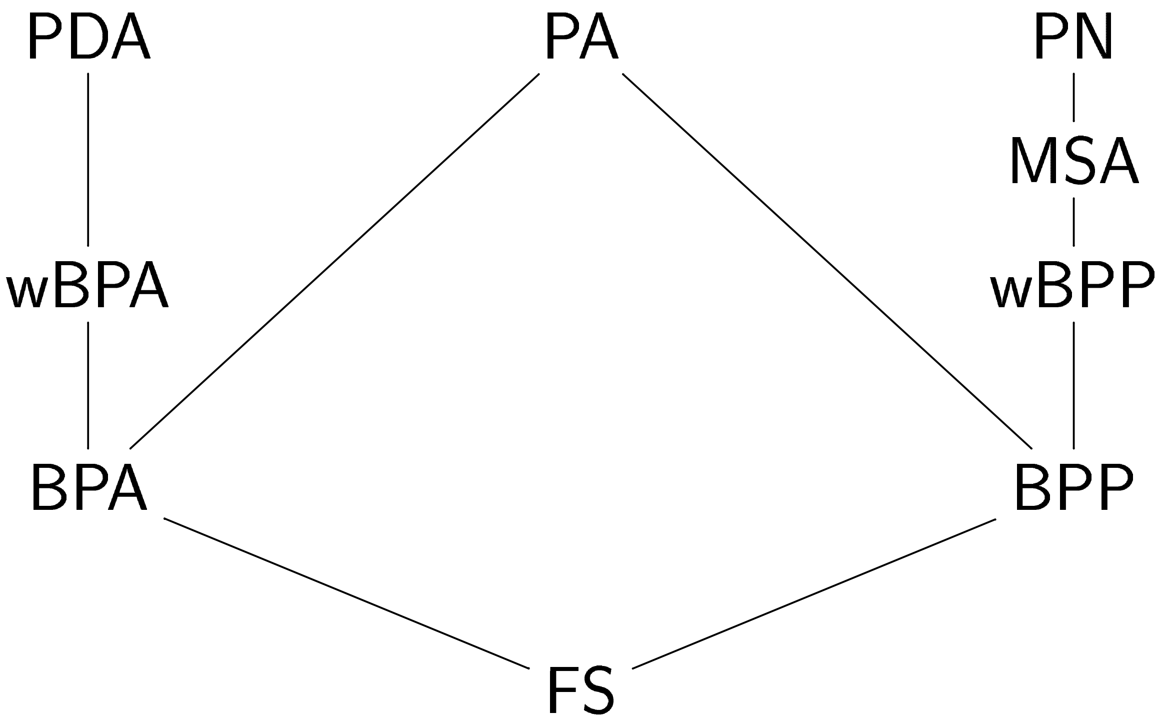
Extended PRS-hierarchy



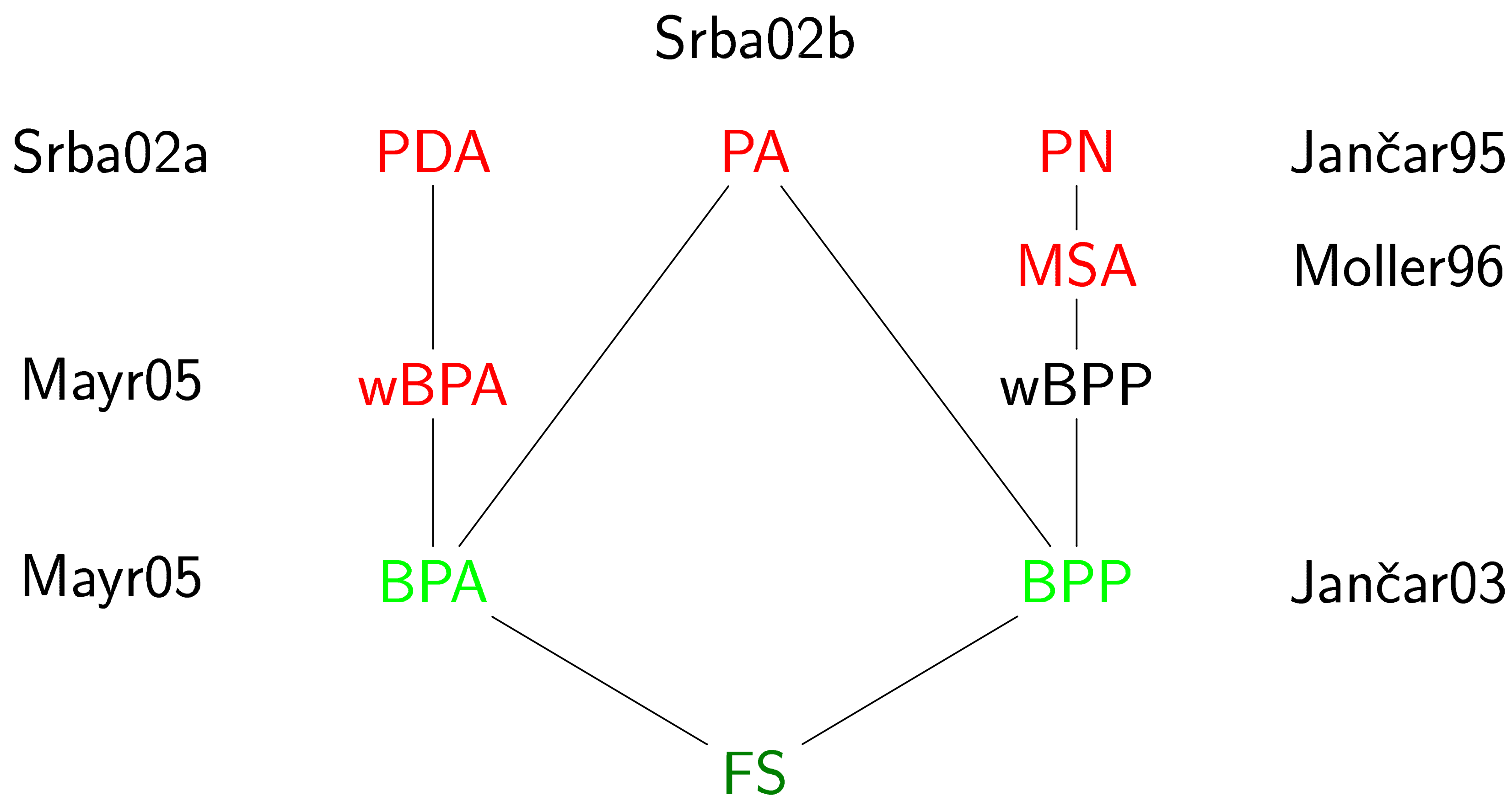
Extended PRS-hierarchy (Abbreviated)



Refined Hierarchy



(Un)decidability of Weak Bisimilarity



The Main Result

Weak bisimilarity is undecidable for wBPP.

The proof is constructed as a reduction of Minsky machine non-halting problem.

Minsky machine M $\Longrightarrow$ wBPP Δ and two of its states s.t.
M does not halt iff the two states are weakly bisimilar

Minsky Machine

$$
\begin{array}{ll}
l_1: & \ldots \\
\vdots & \\
l_i: & inc(c_1);\ goto\ l_r; \\
\vdots & \\
l_j: & if\ (c_2 = 0)\ goto\ l_s; \\
 & \quad else\ dec(c_2);\ goto\ l_r; \\
\vdots & \\
l_n: & halt;
\end{array}
$$

$$
\begin{array}{ccc}
label & c_1 & c_2
\end{array}
$$

$$
\begin{tikzcd}
(\, l_i \,,\, x \,,\, y \,) \arrow[d] \\
(\, l_r \,,\, x{+}1 \,,\, y \,)
\end{tikzcd}
$$

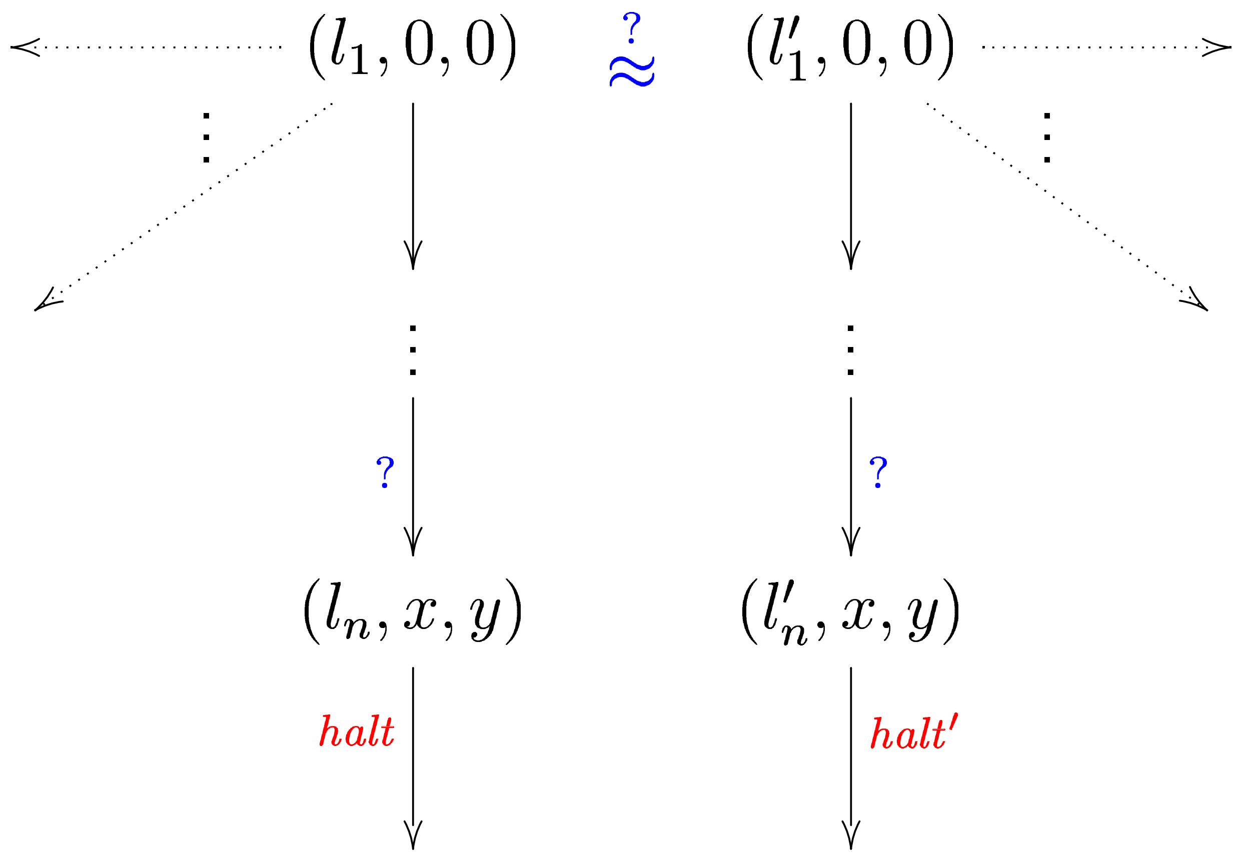
$$
\begin{tikzcd}
 & (\, l_j \,,\, x \,,\, y \,) \arrow[dl] \arrow[dr] & \\
(\, l_s \,,\, x \,,\, 0 \,) & & (\, l_r \,,\, x \,,\, y{-}1 \,)
\end{tikzcd}
$$

Non-halting problem:

Instance: A Minsky machine (list of instructions).

Question: Is the (deterministic) run from $(l_1, 0, 0)$ infinite?

Bisimulation Game



Representation of a MM Configuration

A configuration $(\, l_j \,,\, v_1 \,,\, v_2 \,)$ is represented by a term

$$L_j \| X_1^{x_1} \| Y_1^{y_1} \| X_2^{x_2} \| Y_2^{y_2}$$

where $v_1 = x_1 - y_1$ and $v_2 = x_2 - y_2$.

Increment Action

$$l_j : \quad inc(c_k); \; goto \; l_r;$$

$$\begin{array}{ccc} simL_j \| \beta & \quad & simL'_j \| \beta \\ \downarrow{\scriptstyle inc} & & \downarrow{\scriptstyle inc} \\ simL_r \| X_k \| \beta & & simL'_r \| X_k \| \beta \end{array}$$

Denotation:

Let β be a term of the form $X_1^{x_1} \| Y_1^{y_1} \| X_2^{x_2} \| Y_2^{y_2}$.

Checking Rules

$$check_1 X_1 \xrightarrow{chk_1} check_1 \varepsilon \qquad check_2 X_2 \xrightarrow{chk_2} check_2 \varepsilon$$

$$check'_1 Y_1 \xrightarrow{chk_1} check'_1 \varepsilon \qquad check'_2 Y_2 \xrightarrow{chk_2} check'_2 \varepsilon$$

$$check_k \beta \approx check'_k \beta$$

$$\Longleftrightarrow$$

β contains the same number of process constants X_k as process constants Y_k

$$\Longleftrightarrow$$

the value of c_k represented by β equals zero

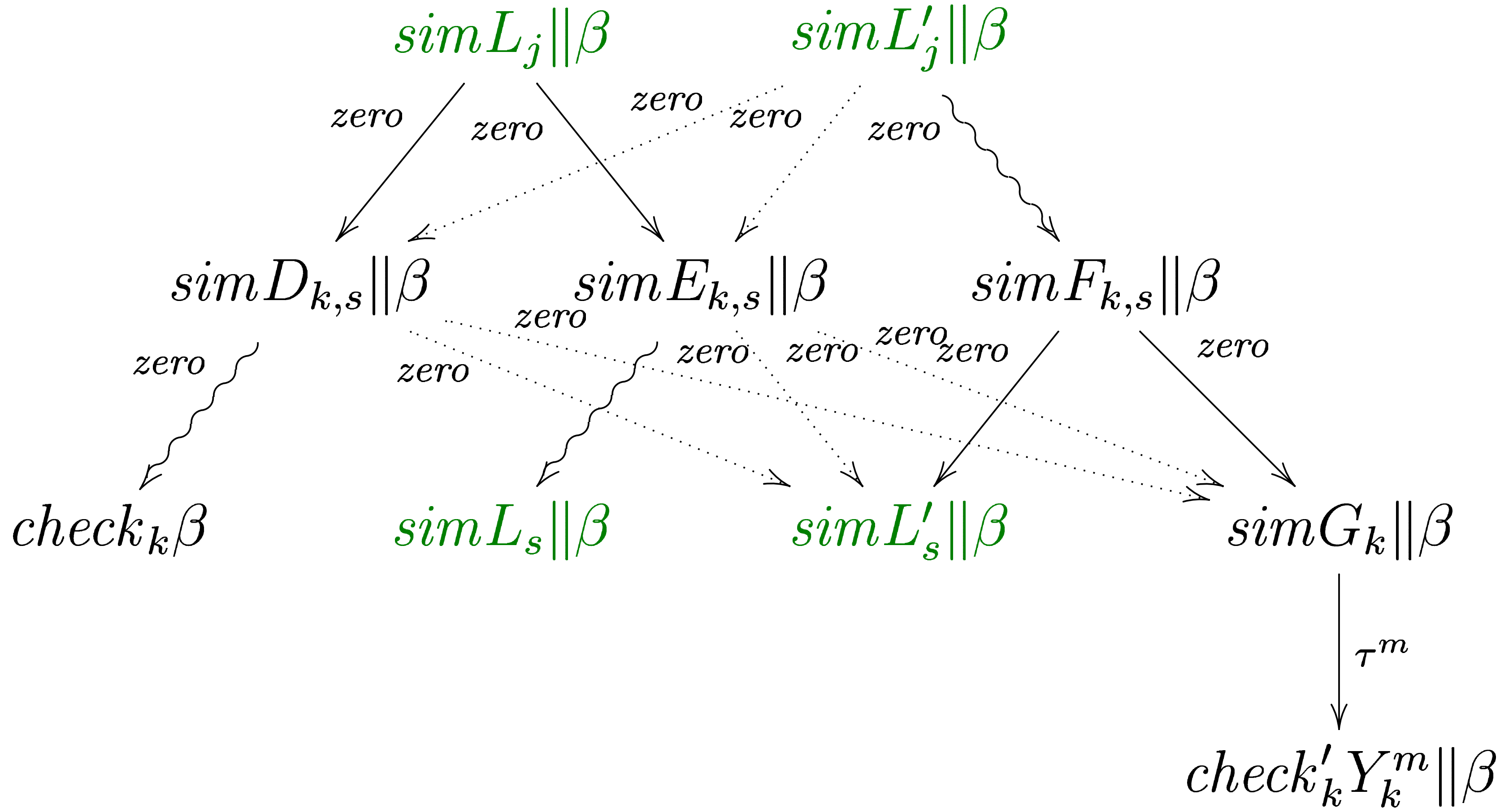
Decrement Action

$$l_j : \quad if\ (c_k = 0)\ goto\ l_s; \quad else\ dec(c_k);\ goto\ l_r;$$

$$\begin{tikzcd}
 & & simL_j \| \beta \arrow[dl, "dec"'] \arrow[dr, "dec"] & & simL'_j \| \beta \arrow[dlll, dotted, "dec"'] \arrow[dl, dotted, "dec"'] \arrow[dr, squiggly, "dec"] & & \\
 & simA_{k,r} \| \beta \arrow[dl, squiggly, "dec"'] \arrow[drrr, dotted, "dec"'] \arrow[drrrrr, dotted, "dec"] & & simB_{k,r} \| \beta \arrow[dl, squiggly, "dec"'] \arrow[dr, dotted, "dec"] \arrow[drrr, dotted, "dec"] & & simC_{k,r} \| \beta \arrow[dl, "dec"'] \arrow[dr, "dec"] & \\
check_k \beta & & simL_r \| Y_k \| \beta & & simL'_r \| Y_k \| \beta & & check'_k \beta
\end{tikzcd}$$

Zero Action

$$l_j : \quad if\ (c_k = 0)\ goto\ l_s; \quad else\ dec(c_k);\ goto\ l_r;$$



Reachability Simple Property

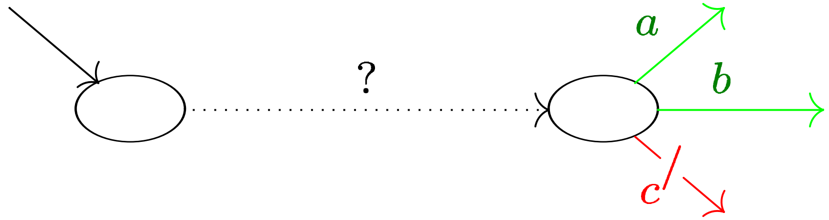
Reachability simple property

$$\phi \quad ::= \quad tt \;\mid\; \neg\phi \;\mid\; \phi \wedge \phi \;\mid\; \langle a\rangle tt \;\mid\; \langle a\rangle\neg tt$$

$$\Delta \models \mathsf{EF}\phi$$

Example:

$$\Delta \models \mathsf{EF}(\ \langle a\rangle tt \ \wedge\ \langle b\rangle tt \ \wedge\ \neg\langle c\rangle tt\ )$$



Reachability HM Property

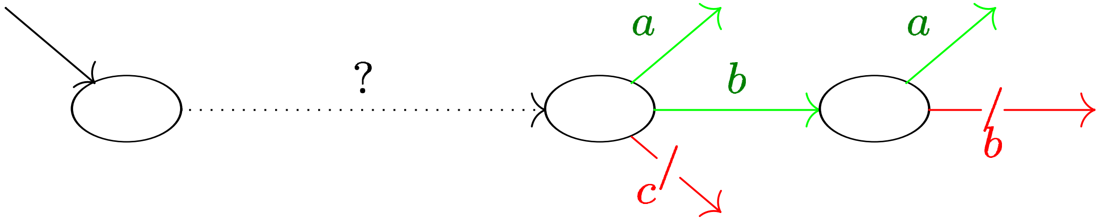
Reachability HM property - nesting of diamonds

$$\phi \quad ::= \quad tt \;\mid\; \neg\phi \;\mid\; \phi \wedge \phi \;\mid\; \langle a\rangle\phi$$

$$\Delta \models \mathsf{EF}\phi$$

Example:

$$\Delta \models \mathsf{EF}(\ \langle a\rangle tt \ \wedge\ \langle b\rangle(\langle a\rangle tt \wedge \neg\langle b\rangle tt)\ \wedge\ \neg\langle c\rangle tt\ )$$



Decidability of EF logic

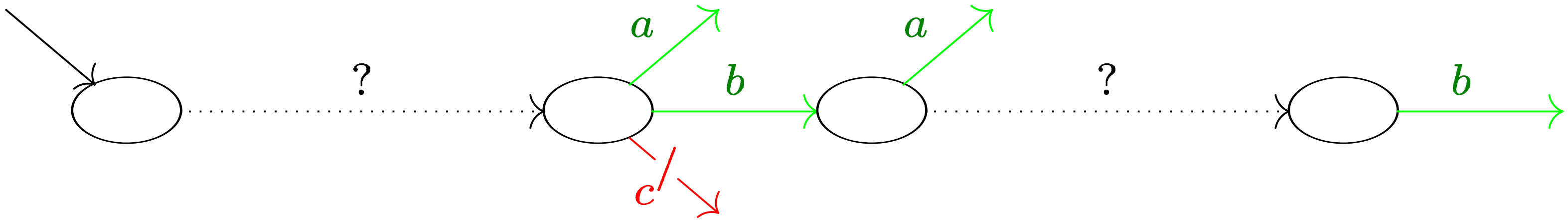
Decidability of EF logic - nesting of EF operators

$$\phi \quad ::= \quad tt \ \mid \ \neg\phi \ \mid \ \phi \wedge \phi \ \mid \ \langle a \rangle \phi \ \mid \ \mathsf{EF}\phi$$

$$\Delta \models \phi$$

Example:

$$\Delta \models \mathsf{EF}(\ \langle a \rangle tt \ \wedge \ \langle b \rangle(\langle a \rangle tt \wedge \mathsf{EF}\langle b \rangle tt) \ \wedge \ \neg\langle c \rangle tt\ )$$



Decidability Problems for EF Logic

Reachability simple property

$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle tt \mid \langle a \rangle \neg tt$$

$$\Delta \models \mathsf{EF}\phi$$

Reachability HM property - nesting of diamonds

$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi$$

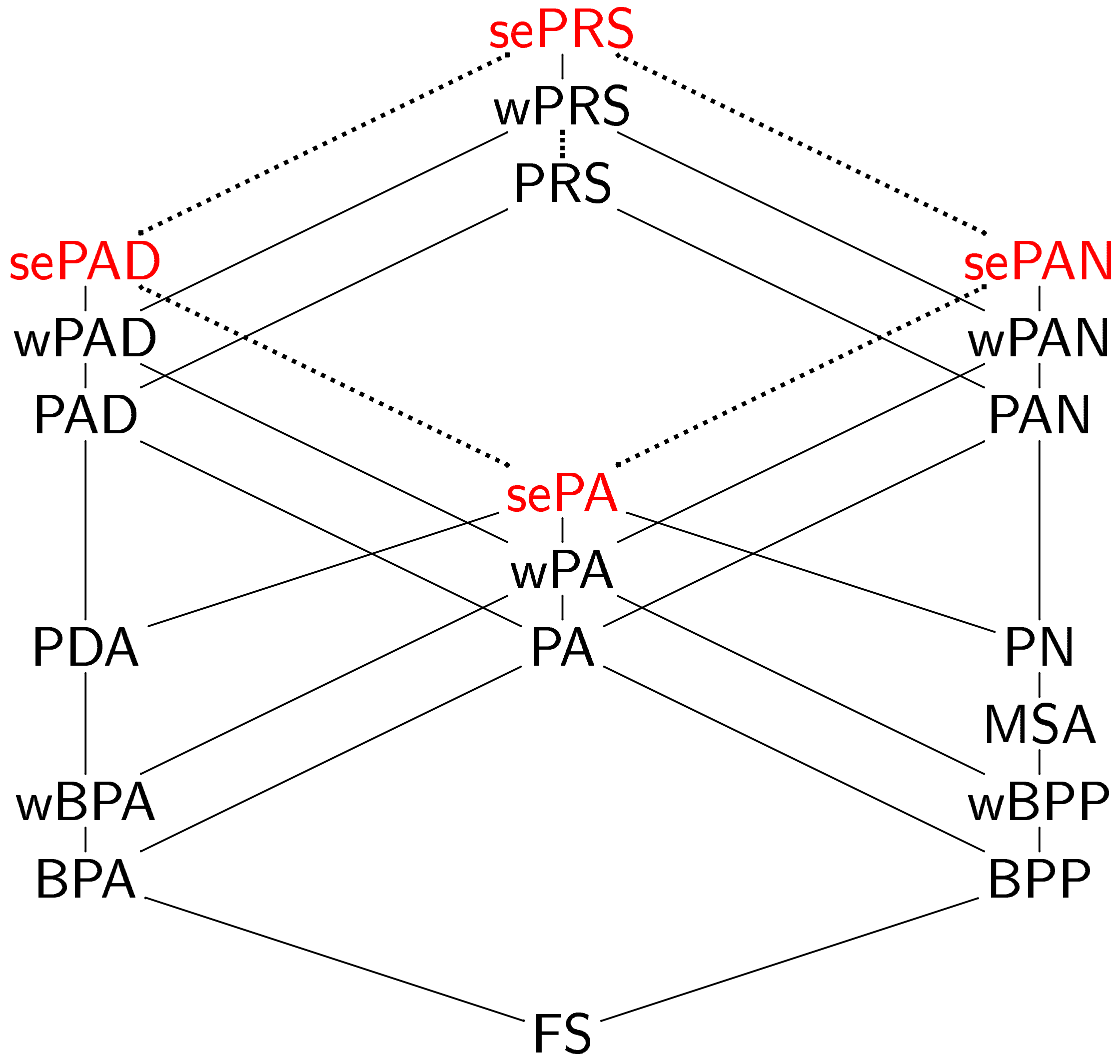
$$\Delta \models \mathsf{EF}\phi$$

Decidability of EF logic - nesting of EF operators

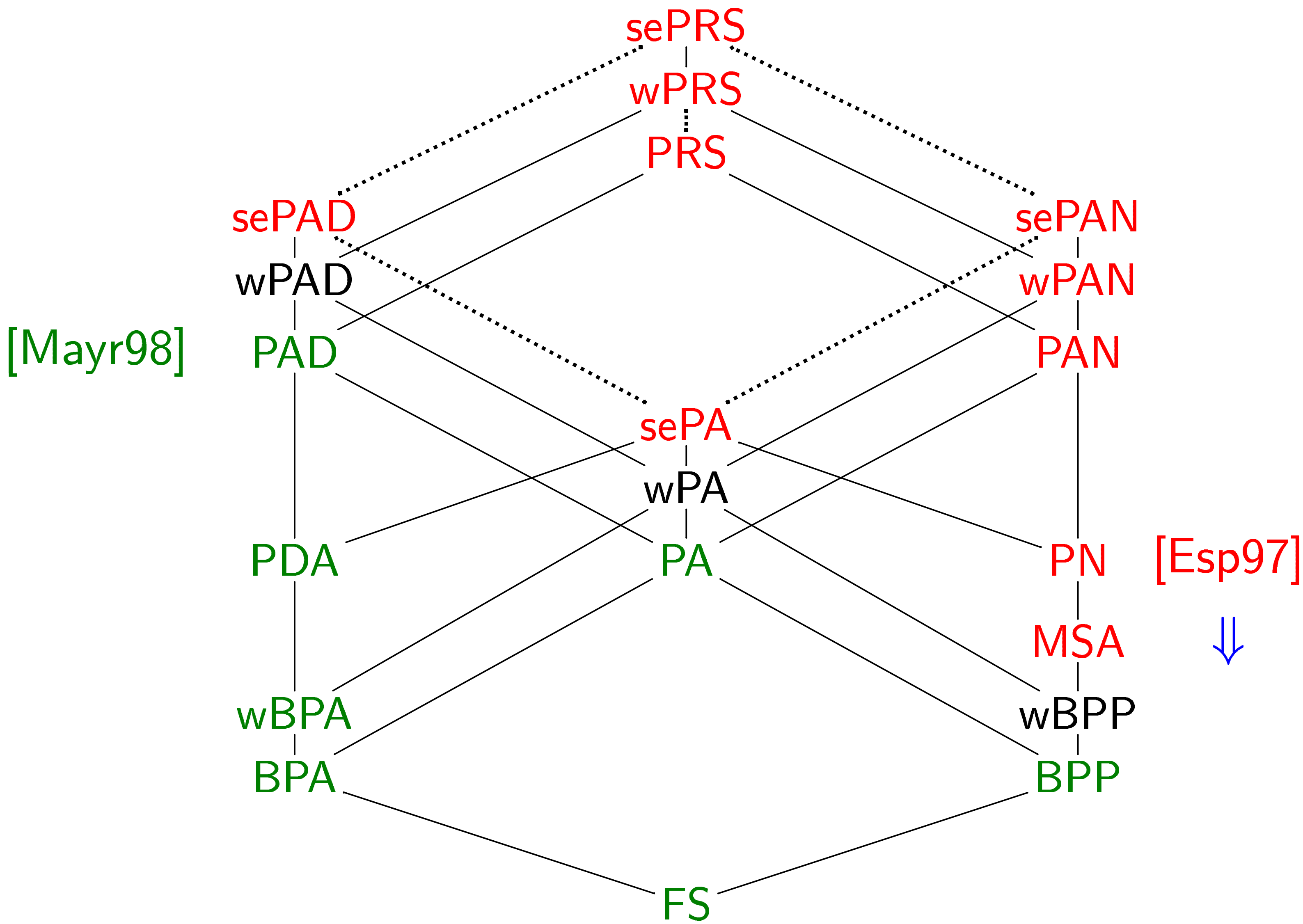
$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi \mid \mathsf{EF}\phi$$

$$\Delta \models \phi$$

Turing Powerfull Classes



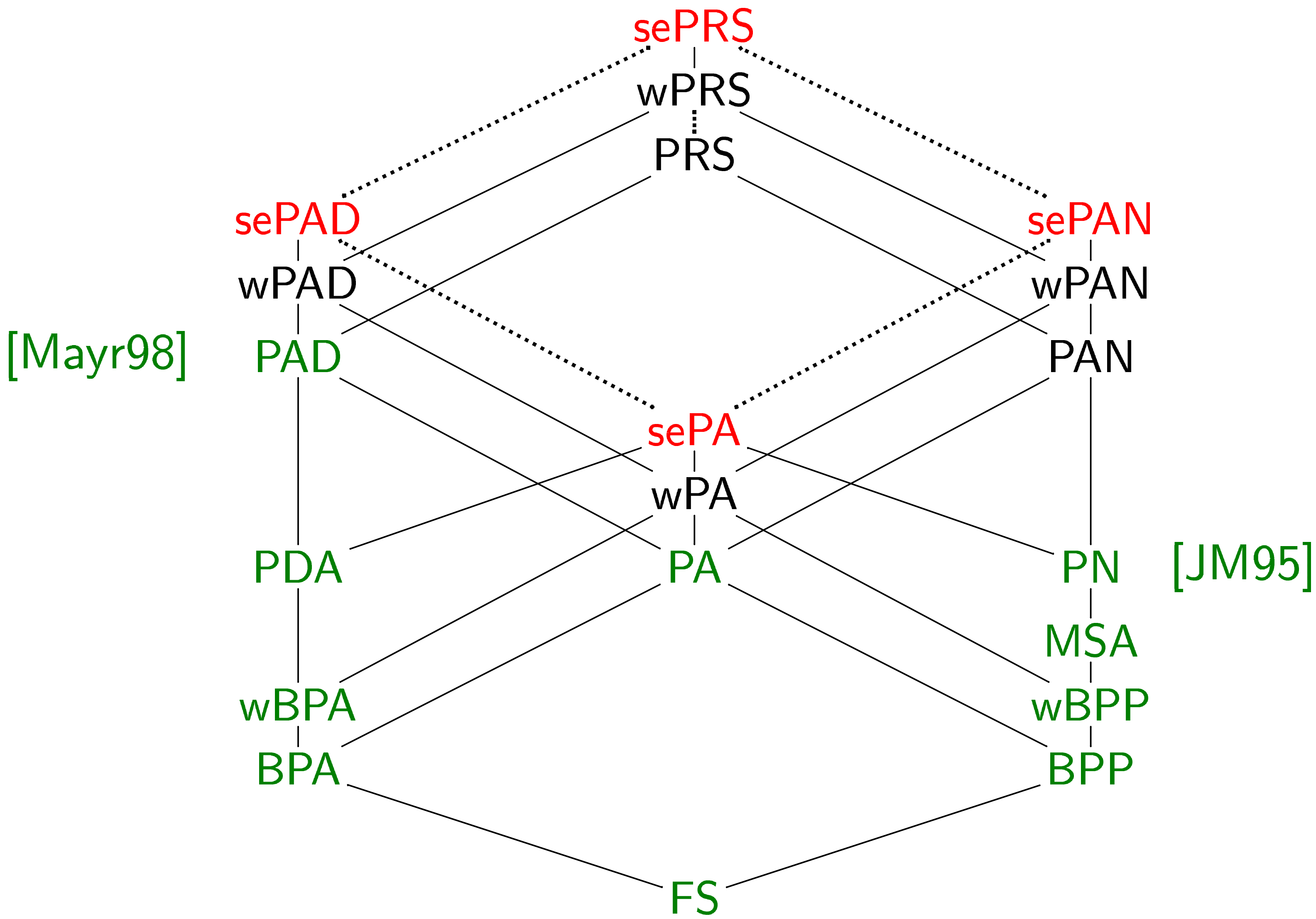
Decidability of EF Logic



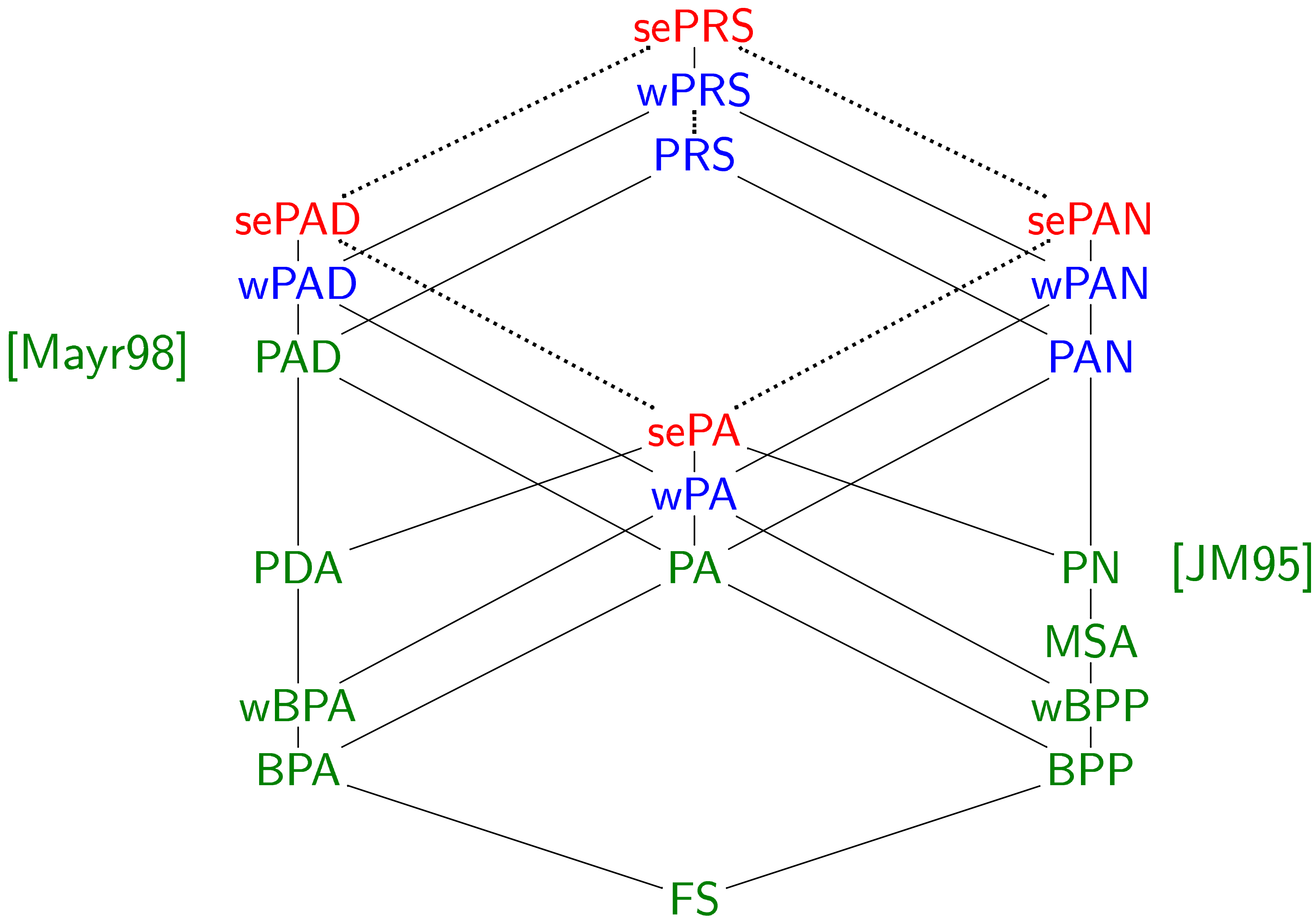
Our Contribution

- We strengthen the undecidability border of EF logic for PN onto MSA=seBPP.

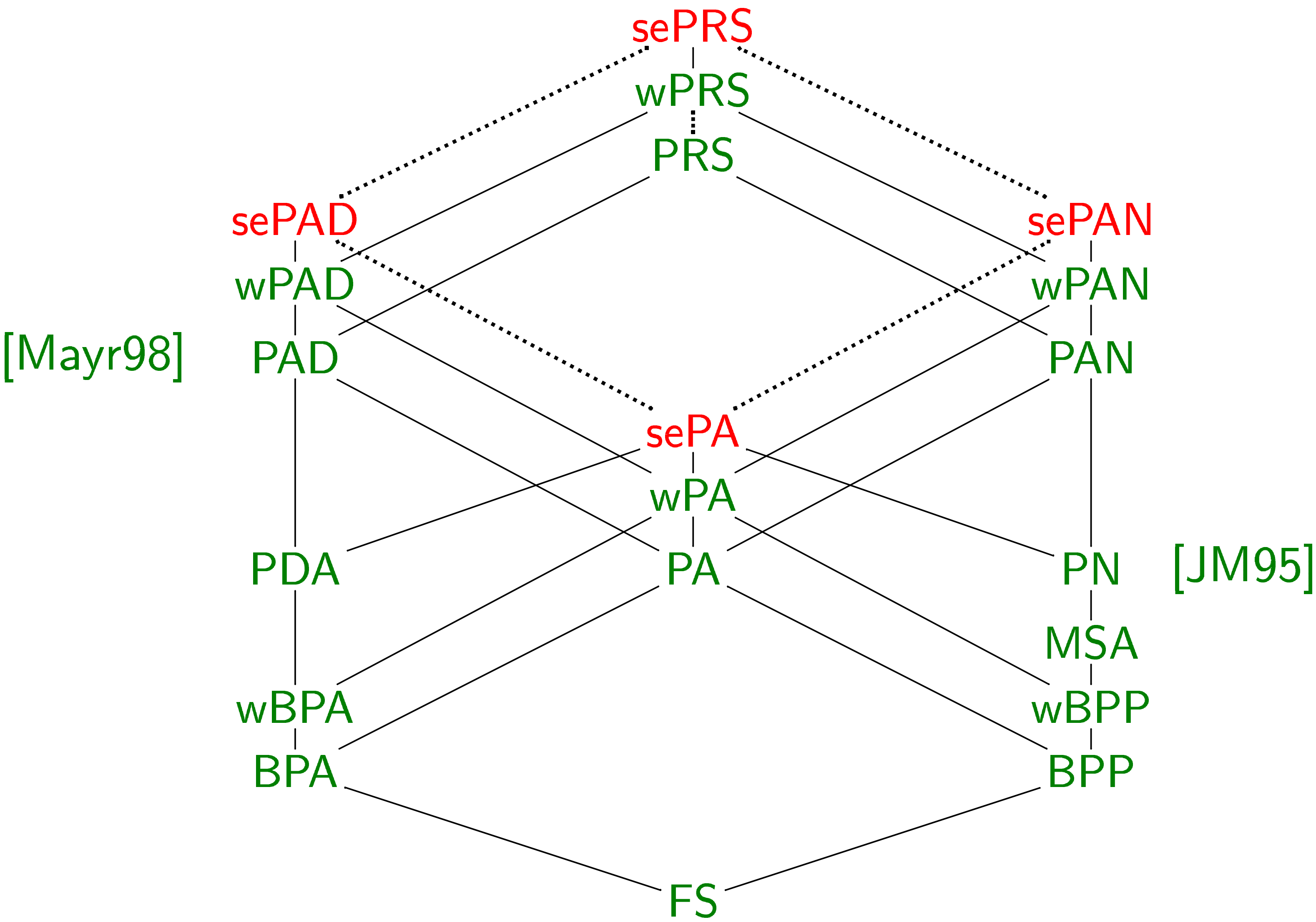
Reachability HM Properties



Reachability HM Properties



Reachability HM Properties



Idea and Implication of the Main Result

Proof:

- constructive reduction onto reachability problem for wPRS [KRS04]

Implication:

- reachability HM property $\Rightarrow$ bisimulation with FS [JKM01]

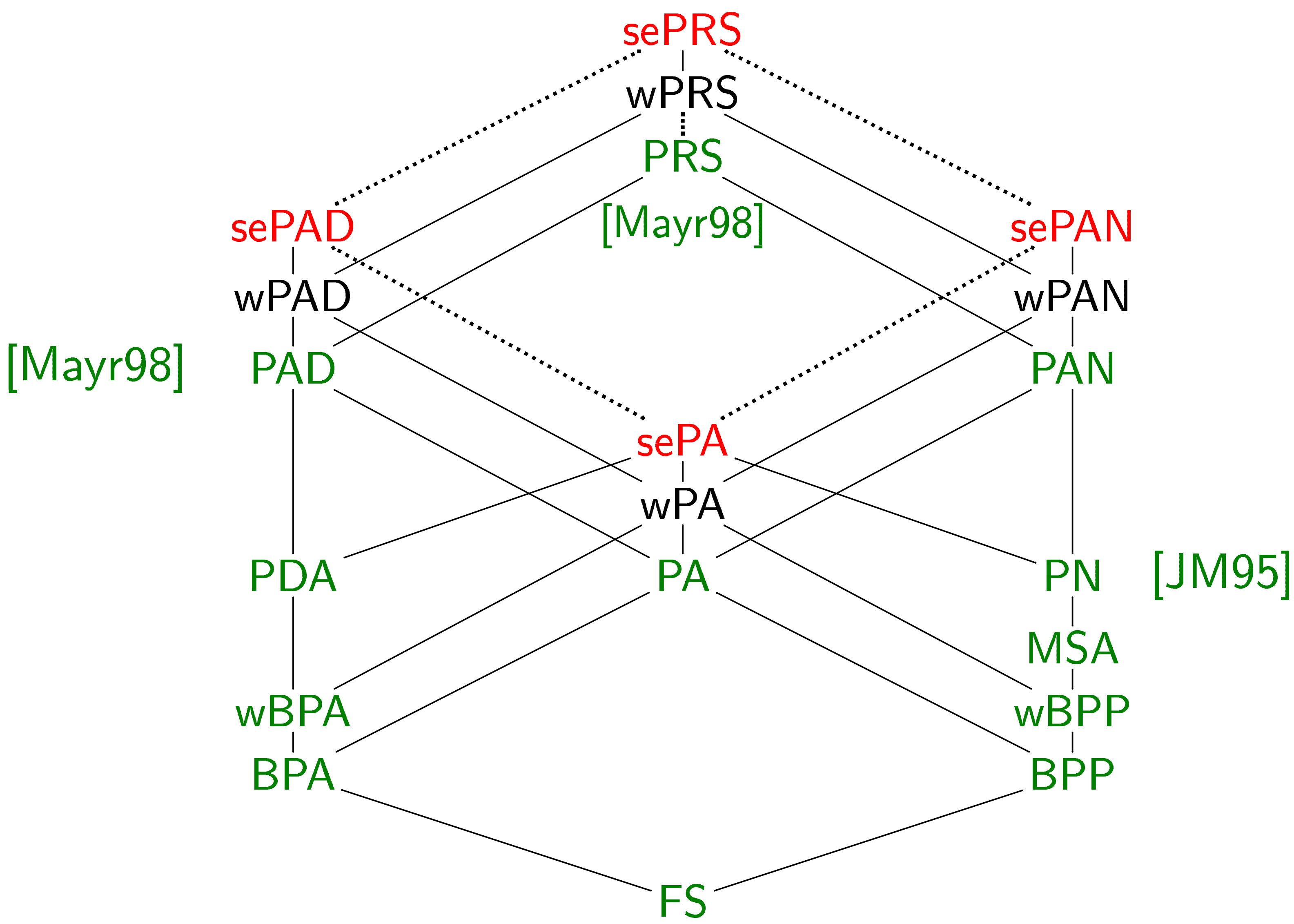
- bisimulation with FS is decidable for PRS and PAN
  open problems formulated in Roadmap of Infinite Results [Srba]

Our Contribution

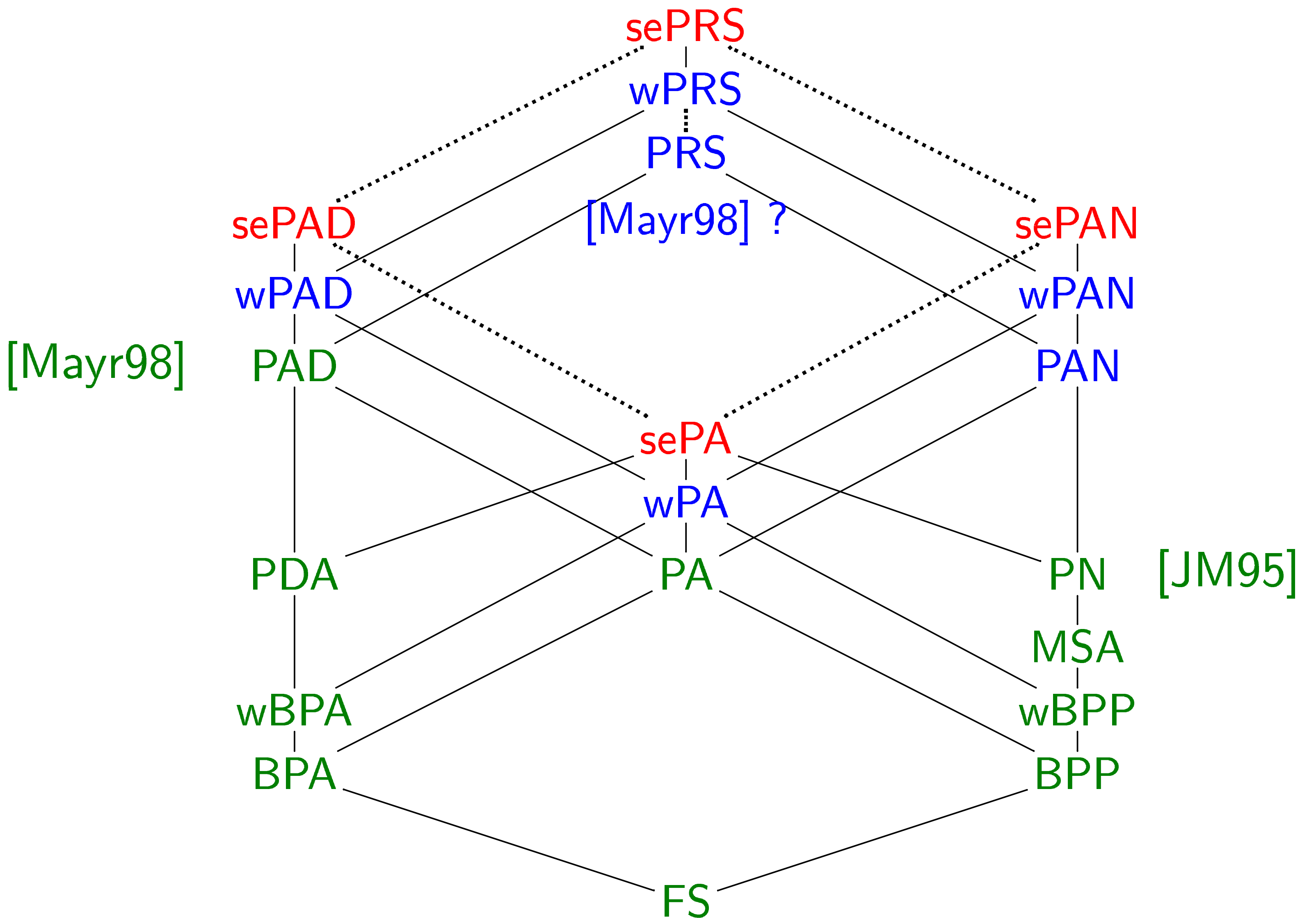
- We strengthen the undecidability border of EF logic for PN onto MSA=seBPP.

- The decidability of reachability HM property is new not only for wPA, wPAD, wPAN, and wPRS but also for the original PAN and PRS.

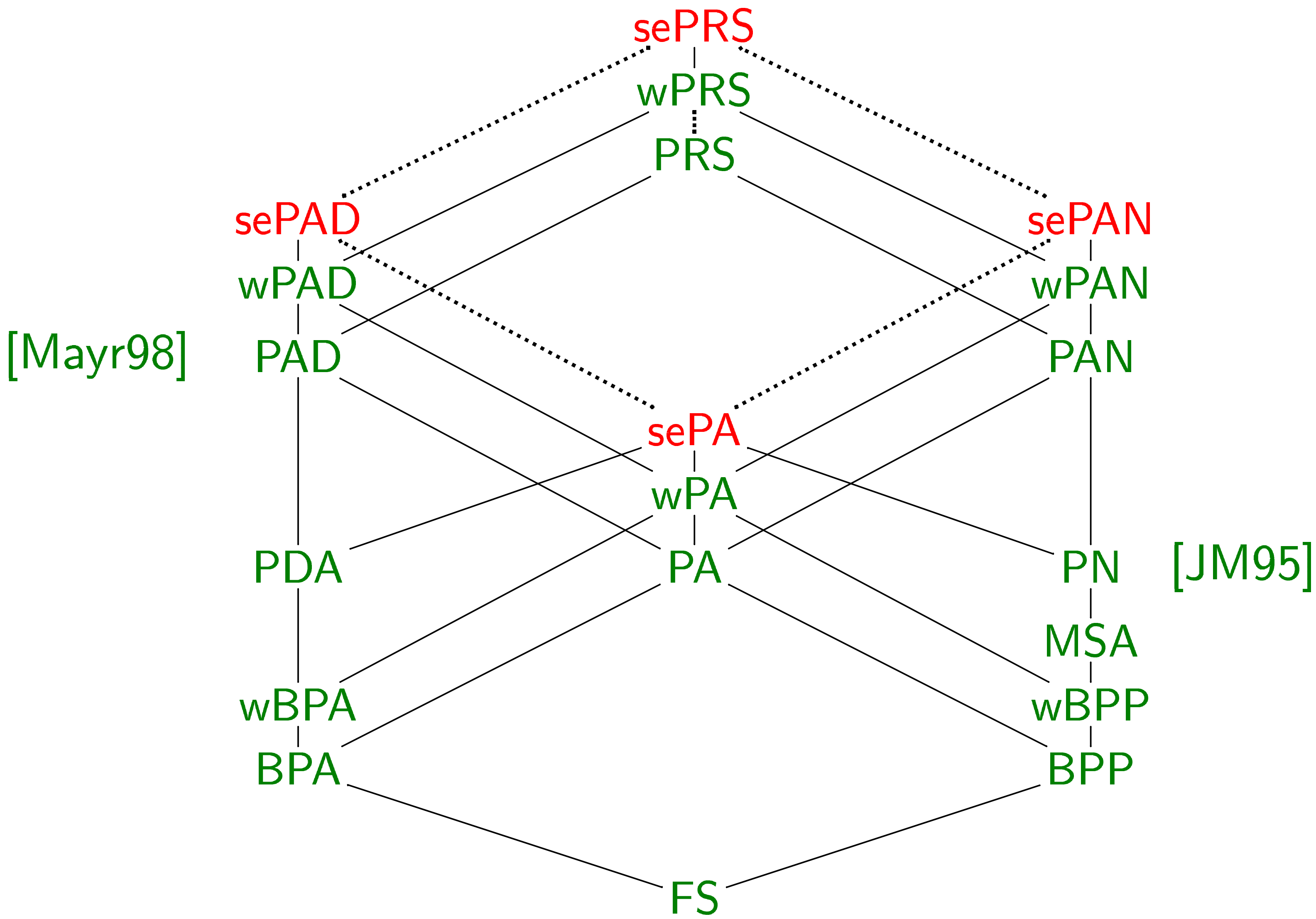
Reachability Simple Properties



Reachability Simple Properties



Reachability Simple Properties



Our Contribution

- We strengthen the undecidability border of EF logic for PN onto MSA=seBPP.
- The decidability of reachability HM property is new not only for wPA, wPAD, wPAN, and wPRS but also for the original PAN and PRS.
- The decidability of reachability simple property is new for wPA, wPAD, wPAN, wPRS, . . .

Summary and Future Work

Summary

- EF logic is undecidable for MSA
- reachability HM property is decidable for wPRS
- bisimulation with FS is decidable for wPRS

Future work

- EF logic for wPAD, wPA, and wBPP
- strong and weak bisimulation equivalence

Decidability Problems for EG Logic

Evitability simple property

$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle tt \mid \langle a \rangle \neg tt$$

$$\Delta \models \mathsf{EG}\phi$$

Evitability HM property - nesting of diamonds

$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi$$

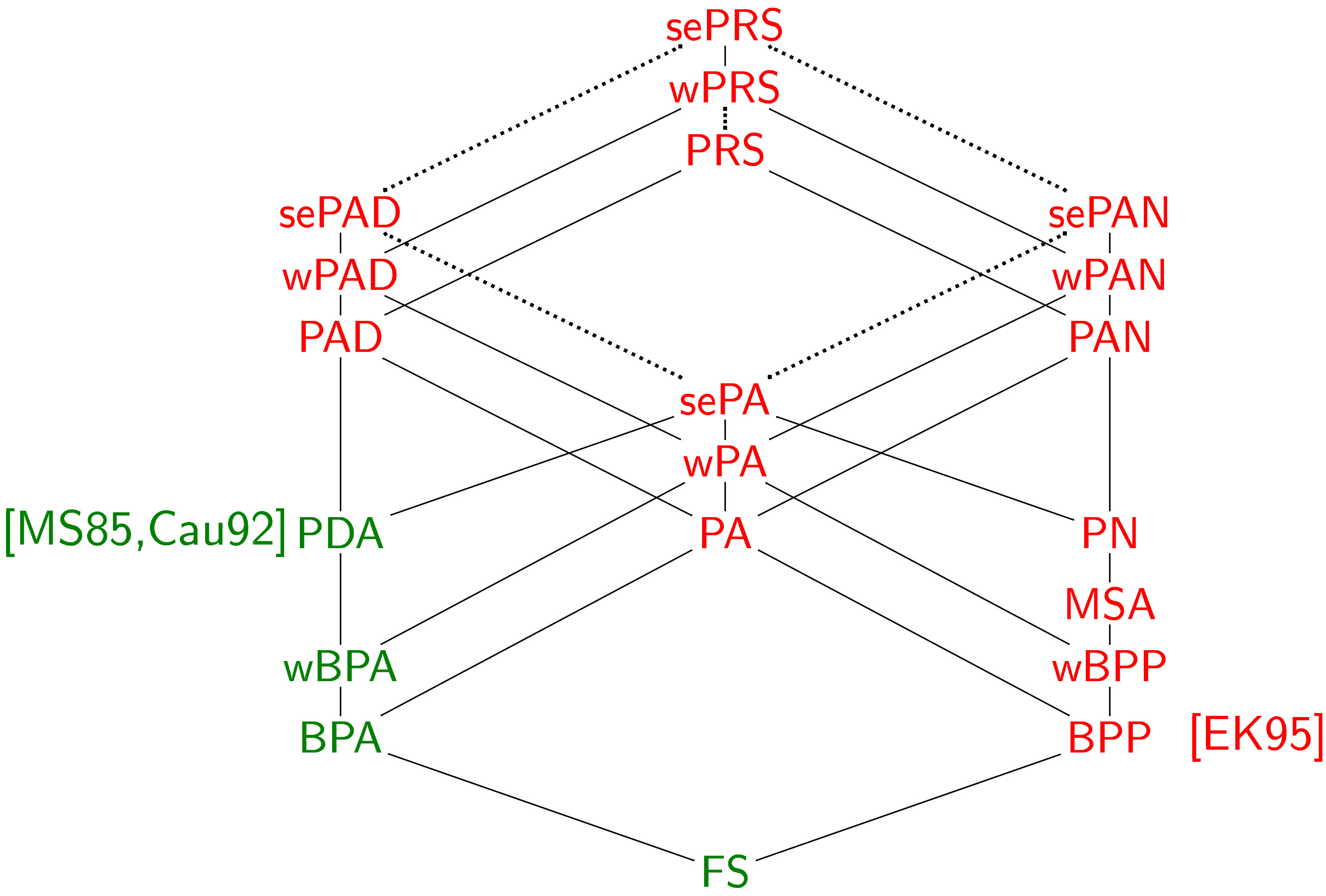
$$\Delta \models \mathsf{EG}\phi$$

Decidability of EG logic - nesting of EG operators

$$\phi \quad ::= \quad tt \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi \mid \mathsf{EG}\phi$$

$$\Delta \models \phi$$

Decidabilities of EG Logics



Summary and Future Work

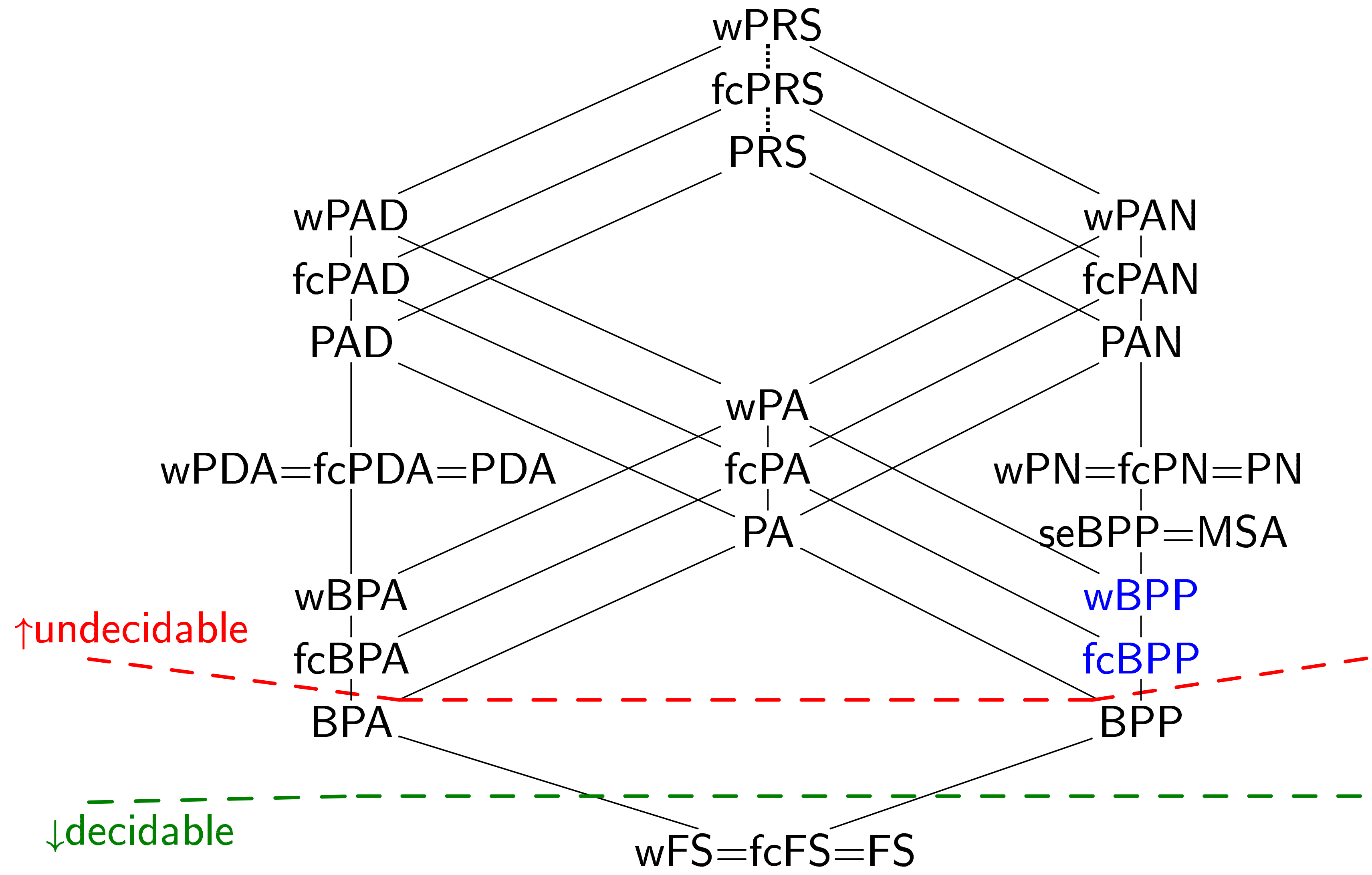
Summary

- EF logic is undecidable for MSA
- reachability HM property is decidable for wPRS
- bisimulation with FS is decidable for wPRS
- evitability simple property is undecidable for BPP

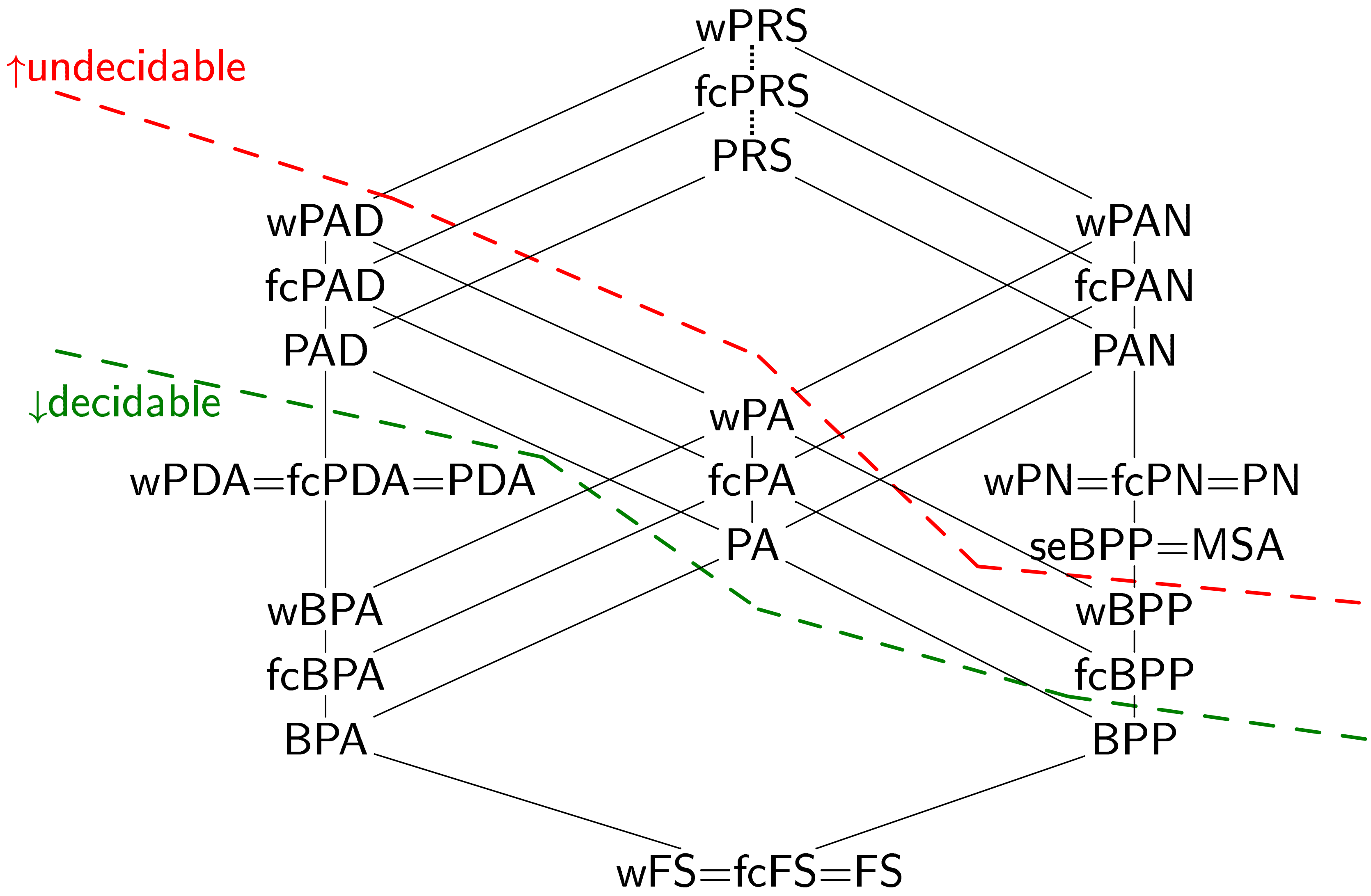
Future work

- EF logic for wPAD, wPA, and wBPP
- strong and weak bisimulation equivalence

Decidability of Weak Bisimulation



Decidability of Strong Bisimulation



Bisimulation Decidability PN – BPP

[Jančar 1995] — PN

[Moller 1996] — seBPP=MSA

↑undecidable

wBPP

fcBPP

↓decidable

[Christensen, Hirsfeld, Moller 1993]
[Jančar 2003] – PSPACE — BPP